Angle of Repose of Selected Bivalve Shell Beds

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ABSTRACT


The angle of repose, $\phi$, is defined as the angle between the tilting bed composed of loose sedimentary material and the horizontal plane when the first faint movement of a particle occurs anywhere on the bed. This angle is required for calculating the critical shear stress for bed erosion. Values of $\phi$, which are not easily available from literature, were determined in laboratory experiments for beds composed of Donax variabilis (coquina), one of Crassostrea virginica (common oyster) and two were composed of mixtures of Chione cancellata (cross-barred chione) and Noetia ponderosa (ponderous ark). The mean value, $\phi$, ranged from 28.40° to 38.60°. An attempt was made to examine the possibility of representing the distribution of $\phi$, or that of simple functions of $\phi$, in the Gaussian form, because such a representation would enable an easily defined statistical description of data on $\phi$. Tests for normality, however, suggested that none of these distributions could be considered to be Gaussian for any of the beds. It is, therefore, recommended that $\phi$ values be characterized by $\phi$, the coefficient of variation of $\phi$, and geometric parameters defining shell size and sphericity. Surface roughness, which is also essential in characterizing $\phi$, must be identified by species, as no easily quantifiable parameter defining roughness is presently available.

ADDITIONAL INDEX WORDS: Angle of repose, bed stability, bivalve shells, erosion, sediment transport, shell transport.

INTRODUCTION

The stability of estuarine beds composed of loosely packed shells is a matter of considerable interest to a variety of sediment-related problems of scientific and engineering significance. For instance, the degree of stability against erosion must be known to determine the rates of sediment transport, and, hence, physical properties of the benthic boundary layer, including suspended sediment concentration and near-bed flow structure. In turn, these properties considerably influence biogenic processes (KIRBY, 1969; KOMAR, 1976; PARCHURE, 1984). In areas where large quantities of shells occur, shell beds have been used to stabilize soil embankments against wave and current attack (LEE, 1978). Similarly, shell beds can be used to prevent the occurrence of scour holes around bridge piers (LEE, 1978). In every case it is necessary to know if the bed will be stable under given flow conditions.

Bed stability is characterized by the critical shear stress for erosion, $\tau_{cr}$, which is equal to the flow-induced bed shear stress, $\tau_{bl}$, at the point of incipient particle motion. If $\tau_{bl} < \tau_{cr}$, the bed will be stable; if $\tau_{bl} \geq \tau_{cr}$, the bed will fail, i.e. erode. The critical shear stress is related to the particle size, $d_p$, according to the well-known Shields' relationship (SHIELDS, 1936):

$$\tau_{cr} = \theta_{cr} (\nu_s - \gamma) d_p$$

(1)

Where $\theta_{cr}$ is referred to as the entrainment...
parameter, $\gamma$, is the unit weight of the particles and $\gamma$ is the unit weight of water. An analytic expression relating $\theta_r$, to bed slope, bed roughness $k$, and the cotangent of the angle of repose, i.e. $\cot \phi$, has been derived and successfully used to predict $\theta_r$, and, therefore, $\tau_r$ (Mehta and Christensen, 1983). In general, $\theta_r$ decreases with increasing slope, increases with $k$, and decreases with increasing $\phi$. The value of $k$, can be calculated from velocity measurements at the site where the bed slope is known (Mehta, 1978). Therefore, if $\phi$ is known, $\theta_r$, can be calculated and $\tau_r$, determined from equation (1). Laboratory experiments were therefore conducted in a specially designed apparatus to measure $\phi$ for four beds of bivalve shells which are commonly found in Florida.

**ANGLE OF REPOSE AND INCIPIENT MOTION**

According to a commonly used definition (Van Burkalow, 1945), a “natural slope,” or slope of repose, may be obtained by: (1) dropping loose material from above to form a conical pile, or (2) removing support from the edge of a pile of material so that the fragments fall of their own weight over a free edge until the slope of the pile becomes critical, and reaches the angle of repose, i.e. the angle between this slope and the horizontal plane. However, in terms of the condition for bed failure, the definition of $\phi$, is generally understood in literature on cohesionless sediment transport to be the one corresponding to the first faint movement of a particle anywhere on the bed (Graf, 1971). This is the present case because of the need to determine, for stable bed design, the minimum value of the bed shear stress for erosion (i.e. the critical shear stress) anywhere on the bed as opposed to that for a particular particle. The bed may be inclined or horizontal. When the bed is inclined, but the slope is less than critical, failure due to flow will occur more easily than when it is horizontal, because in the former case, the component of particle weight in the direction of flow brings about failure at a lower value of the critical shear stress than when the bed is horizontal.

The second definition of $\phi$, which is of interest here, thus becomes closer to one for the angle of kinetic or sliding friction (Meriam, 1959). Nevertheless, in keeping with customary usage in cohesionless sediment transport (Graf, 1971; Vanoni, 1975), the term angle of repose will be used to imply the second definition, rather than the one associated with the critical slope of a pile. In fact, since the pile angle is the larger of the two, it yields a larger than actual value of the critical shear stress and therefore is not suitable for calculating this stress.

Van Burkalow (1945) measured the pile angle of numerous materials. Although she considered the pile angle as the angle or repose, a significant conclusion of the investigation was that the angle varied directly with particle size and with surface roughness. Shells were, however, not included in her study.

Lane (1955) presented relationships between the angle of repose and particle diameter for non-cohesive materials ranging from “very round” to “very angular.” The selected definition of particle diameter was not specified. Shells are not mentioned in the study. For particle size ranging from 0.8 cm to 10 cm, the angle varied from about 22° to 41°, depending both upon size and roundness, with the angle increasing with size and decreasing with increasing roundness.

Miller and Byrne (1966) reported measurements of the “angle of repose” for a single grain on an otherwise fixed bed with different glued roughnesses. Grains consisting of glass spheres, natural beach sand and crushed quartzite were tested for stability by slowly tilting the bed, both under immersed (in water) and dry conditions. The reported values of the angle ranged from 28.7° to 90.4°. For loose sedimentary beds angles greater than 50-60° seldom occur, and the probability of attaining a 90° angle, i.e. a vertical bed, without failure, is nil. As Miller and Byrne have noted, their measurements did not correspond with the “mass” angle of repose of loose beds. An examination of their data shows that, on the average, values based on measurements under immersed condition exhibited no significant difference from those measured in the dry state. The primary reason for this observation appears to be that the smallest grain diameter was 0.175 mm, so that cohesion was not important. In accordance with this observation, the present experiments were conducted in the dry state, since immersing shells in water would have required more elaborate and time-consuming tests.

**METHODOLOGY**

Experimental observations were made relative to $\phi$, and in one experiment two additional successively increasing angles, $\phi_1$ and $\phi_2$, with respect to the horizontal, were measured as well. The value $\phi$ was considered to be the desired angle when the bed, whose inclination was slowly increased from the horizontal, showed the first faint, visually observed,
parameter, $\gamma$, is the unit weight of the particles and $\gamma$ is the unit weight of water. An analytic expression relating $\theta_r$ to bed slope, bed roughness $k$, and the cotangent of the angle of repose, i.e. $\cot \phi$, has been derived and successfully used to predict $\theta_r$, and, therefore, $\tau_{cr}$ (Mehta and Christensen, 1983). In general, $\theta_r$ decreases with increasing slope, increases with $k$, and decreases with increasing $\phi$. The value of $k$, can be calculated from velocity measurements at the site where the bed slope is known (Mehta, 1978). Therefore, if $\phi$ is known, $\theta_r$ can be calculated and $\tau_{cr}$ determined from equation (1). Laboratory experiments were therefore conducted in a specially designed apparatus to measure $\phi$ for four beds of bivalve shells which are commonly found in Florida.

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movement of a single shell. The value $\phi_1$ was the bed angle when two to three shells began to slide simultaneously anywhere on the bed. Finally, $\phi_2$ was the bed angle corresponding to a mass slide of a group of shells consisting of more than two to three shells, locally. The angles $\phi_1$ and $\phi_2$ clearly have a more qualitative meaning than $\phi$. However, there were two reasons for including measurements of these two angles: (1) at least in visual terms, three selected states of bed stability would be quantified, and (2) if it was found that $\phi$ was measurably lower than $\phi_1$, then the importance of selecting $\phi$ for a conservative estimation of bed stability would be emphasized. Furthermore, the angle $\phi_2$ was likely to be close to the angle which would be obtained from a loose pile.

The spatial distributions of shell attitude and interlocking frictional forces between shells in the surficial layer of the bed are contingent upon shell size, shape (including sphericity and roundness), surface roughness (sculpture), sorting and the packing arrangement (compaction). The characteristics of a large bed are best described mathematically using a stochastic approach. A variable $x$, which represents a characteristic of individual shells in a large bed, is expressed as

$$x = \bar{x} + \sigma_x z_x$$  \hspace{1cm} (2)

where $\bar{x}$ and $\sigma_x$ are, respectively, the mean and the standard deviation of the $x$-values for the entire bed. The variable $z_x = (x - \bar{x})/\sigma_x$ is then a random variable with a probability distribution determined by the distribution of the values of $z_x$ in the entire bed. A complete stochastic description of $x$ is provided by specifying the values of $\bar{x}$ and $\sigma_x$ (or the coefficient of variation, $s_x = \sigma_x/\bar{x}$) as well as the probability distribution of $z_x$.

Since there exists a wide variety of techniques for describing and analyzing measurements from a normal (Gaussian) population, it is natural to ask whether $\phi$ or some simple function of $\phi$ could be regarded as normal. Because $s_x$ varies with $\cot \phi$, as noted, one such function is $\cot \phi$. For a loose bed the value of $\phi$ is bounded between $0^\circ$ and $90^\circ$, so that the value of $\cot \phi$ is bounded between 0 and $+\infty$. Because both $\phi$ and $\cot \phi$ take only positive values, theoretically, at least, these variables cannot be normal, although a good normal approximation may be possible if the standard deviations are small relative to the mean.

For a positive variable $x$, the representation

$$x = \exp(\bar{x} + \sigma_x z_x)$$  \hspace{1cm} (3)

where $z_x$ has mean of 0 and standard deviation of 1 is more appropriate than the representation given by equation (2). The variable $x$ defined by equation (3) takes only positive values and implies a multiplicative "error" model. Furthermore, from equation (3) it also follows that $\log x = \bar{x} + \sigma_x z_x$, so that $\bar{x}$ and $\sigma_x$ are, respectively, the mean and the standard deviation of $\log x$. Thus for a positive random variable $x$, $\log x$ is an obvious candidate (function) for examination. Accordingly, in addition to $\phi$ and $\cot \phi$, $\log \phi$ and $\log(\cot \phi)$ were also examined for normality. The ranges of values of $\log \phi$ and $\log(\cot \phi)$ are, respectively, $(-\infty, 4.5)$ and $(-\infty, +\infty)$. The same considerations as above apply to the angles $\phi_1$ and $\phi_2$.

### APPARATUS, MATERIAL AND PROCEDURE

**Apparatus**

The experiments were performed in an apparatus shown in Figure 1. It consisted of a 1 m long, 0.23 m wide and 0.6 m high plexiglass container open at the top. The container rested on a slightly longer and wider aluminum support. This support was hinged at one end to a horizontal aluminum base of the same horizontal dimensions as the support. The base was accurately positioned on a steel table with the help of a spirit level. The other end of the container support could be lifted slowly and smoothly by turning the handle of an Archimedes screw arrangement. The appropriate angle between the horizontal base and the inclined support was measured with the help of two vertical, linear, vernier scales (not shown in Figure 1) which were placed a known horizontal distance apart.

![Figure 1. Apparatus used for measuring angles of stability.](image-url)
Material

Four types of bivalve shells commonly found in Florida were selected. The species were: *Donax variabilis* (coquina), *Chione cancellata* (cross-barred chione), *Noetia ponderosa* (ponderous ark), and *Crassostrea virginica* (common oyster). Coquina clams were collected from a beach south of Matanzas Inlet on the northern Atlantic coast. Oysters were obtained from bank deposits along the Intracoastal Waterway near Marineland, shown in Figure 2, in the vicinity of Matanzas Inlet. The maximum natural slope of the deposits shown in Figure 2 was found to be in the range of 40°-45°. The unit weights of these carbonate-rich materials in the dry state was found to be approximately 2.75 g/cm² (Lee, 1978). Views of the external shell surfaces of the four species are shown in Figures 3a and 3b. In the natural habitat (Captiva Island beach on the lower Gulf of Mexico coast) from which chione and ark shells were collected, these shells occurred in a mixture composed of approximately equal numbers of the two species. The geometry of these shells is described below separately for chione and ark. However, the beds used in the experiments were composed of mixtures of the two species. In all species except chione, the naturally occurring proportion of left- and right-handed valves was approximately equal. Likewise, the ratio of left- to right-handed valves for chione was 3:1.

Shell Geometry

Various morphometric particle descriptions for sphericity and roundness are used in granulometric analyses (Schreiber, 1978). As a result of the peculiar convex shape of bivalve shells, roundness has a different meaning in this case than in the case of "solid" particles. The specification of the selected shells in terms of the species will be considered here to be sufficient for a description of "roundness" as well as sculpture. Shell dimensions and sphericity are described in the following paragraphs.

Valve shape is conveniently specified in terms of three mutually perpendicular axes a, b, and c, where a is the longest dimension, b is the middle dimension, and c is the shortest dimension. Mehta, Lee, and Christensen (1980) showed that a useful
Figure 3. (a) Views of the surface of three selected bivalve shells: (from left) ponderous ark, cross-barred chione, and coquina. (b) View of the surface of common oyster valve selected for the study.
descriptor of shell sphericity, $\beta$, is given by

$$\beta = \left[ \frac{c}{(ab)^{\alpha_2}} \right] \frac{d_n}{d_s}$$

(4)

The diameter, $d_n$, is defined as

$$d_n = \alpha_2 \left( \frac{A}{\pi} \right)^{\alpha_2}$$

(5)

Where $A$ is the area of the "base" of the shell, i.e., the area projected on the plane formed by the axes $a$ and $b$, and $\alpha_2$ is a geometric coefficient. For a given species of valves of different sizes, $\alpha_2$ was found to be relatively invariant (Mehta and Christensen, 1977). The diameter, $d_n$, in equation (4) is the nominal diameter of a sphere having the same volume as the shell. The ratio $c/(ab)^{\alpha_2}$ is recognized as the well-known Corey shape factor (Vanoni, 1975). For a sphere $\beta = 1$, and its value becomes smaller as the particle becomes flatter.

In Table 1 the mean values of $a$, $b$, $c$, $\alpha_2$, $d_n$, and $\beta$ are given for the selected shell species. These statistics are based on representative sub-samples of 30 in the case of coquina, chione, and ark, and 10 in the case of oyster. Procedures for the determination of these parameters, including $d_n$, have been given elsewhere (Lee, 1978; Mehta, Lee, and Christensen, 1980).

### Shell Beds

Four types of beds were constituted. The bed type, mean diameter $d_s$, coefficient of variation, $s_{d_s}$, of the diameter and the mean shape parameter, $\beta$, are given in Table 2. The naturally occurring mixture of chione and ark had a relatively wide size distribution. As a result, the collected sample was divided into two sub-samples, CAI and CAII. This division did not significantly alter the numerical fractions of the two species in the two sub-samples from the fractions in the original sample. However, the size distributions became different. The $\beta$ values are assumed to be the average $\beta$ of chione and ark. The coefficient of variation of $\beta$, i.e., $D_\beta$, was found to be less than 6% for all four beds (Lee, 1978).

### Procedure

Each shell bed was initially prepared while the container rested horizontally. Shells were added manually in small groups in as random a manner as possible, until an approximately horizontal bed of 20 mm to 30 mm thickness was formed. The support was then tapped gently with a mallet until a more horizontal bed resulted. Each test consisted of tilting the bed slowly until incipient particle motion was observed visually. The value of $\phi$ was calculated from

$$\phi = \tan^{-1} \left( \frac{z_2 - z_1}{\ell} \right)$$

(6)

where $z_1$ and $z_2$ are elevations of the support at two points separated by a horizontal distance $\ell$. In those tests in which $\phi_1$ and $\phi_2$ were also measured, tilting of the bed was continued after the observation for $\phi$, until measurements for $\phi_1$ and $\phi_2$ were made, in that order.

The number of measurements in each experiment is given in Table 3. The coquina bed (C) was used in two experiments. In the first experiment (C-1), 30 measurements were made each for $\phi$, $\phi_1$ and $\phi_2$. In all the remaining experiments, only $\phi$ was measured.

### RESULTS

Data from the five experiments were analyzed to test for normality of the four functions described earlier. For experiments C-1 and O, the Shapiro-Wilk (Shapiro and Wilk, 1965) W-test was used inasmuch as this test is known to possess good power properties when the sample size is less than 50. For experiments C-2, CAI and, CAII, where the sample size was 100, the modified form of the Kolmogorov-Smirnov (Stephens, 1974) D-test was used.

Large values of $W$ and small values of $D$ indicate
that a normal distribution fits the data well. A statistical basis for accepting the normal distribution as the appropriate distribution is provided by the value of the level attained by the test: \( \alpha = \) the probability (Pr) of obtaining a value of W (D) smaller (larger) than that calculated from the data. While larger values of the attained level suggest better fit of the normal distribution, a value \( \alpha \geq 0.15 \) is usually considered essential for justifying the assumption of normality (RAO et al., 1979). The Univariate Procedure of the SAS computer package (HELWIG and COUNCIL, 1979) was used to calculate the values of D, W, and \( \alpha \). Table 4 provides the values of D and \( \alpha \) for experiments C-1 and C-2, while Table 5 contains the values of D and \( \alpha \) for experiments C-1, CAI, and CAII.

An examination of Table 4 reveals that the log and cot functions of \( \phi \) for oyster and all functions of \( \phi_1 \) for coquina had normal distributions. However, owing to the fact that the values in Table 4 are based on only 30 measurements, it would be interesting to see whether experiments with larger sample sizes will also support the hypothesis of normal distribution. Table 4 also indicates that for coquina, neither functions of \( \phi \) nor functions of \( \phi_2 \) had normal distributions. However, results in Table 5, which suggest the possibility (\( \alpha = 0.102 \)) of a normal distribution for cot\( \phi \) for coquina (see Figure 4) may appear to contradict the conclusions from the values in Table 4. An analysis of the coquina data for \( \phi \) obtained by pooling results from C-1 and C-2 also led to the conclusion that normal distribution is not appropriate for any of the four functions of \( \phi \).

In view of normal distributions for functions of \( \phi \) for oyster and \( \phi_1 \) for coquina, the nor-normality of the distributions of the functions of \( \phi \) for coquina might appear to counter intuition. An explanation of this discrepancy could be based on the relative sizes of coquina and oyster and their effect on the measurements of \( \phi \). Because coquina are much smaller than oyster (see Table 1), visual detection of first faint movement of a single shell was more difficult for coquina than for oyster. Thus measurement of \( \phi \) for coquina were most likely to be affected by observer bias, which in turn might have caused the distributions to be non-normal.

The normal distribution of \( \phi_1 \) and the non-normal distribution of \( \phi_2 \) could also be explained in a similar manner. The angle \( \phi_1 \) was easier to measure, and was therefore subject to less observer bias. Thus, it is not surprising that the distribution of \( \phi_1 \) for coquina was similar to the distribution of \( \phi \) for oyster. The angle \( \phi_2 \), on the other hand, is an "extreme" angle and was difficult to measure accurately.

The mean and the coefficient of variation (in percent) of the angle of repose for the five experiments are presented in Table 6. Values for the angles \( \phi_1 \) and \( \phi_2 \) for experiment C-1 are also given. The mean, \( \phi_1 \), was 19% higher than \( \phi \) and \( \phi_2 \), which was 31% higher than \( \phi \) in the C-1 experiment with coquina. When bed stability in terms of incipient particle motion is of interest, it is clearly important to use \( \phi \) rather than \( \phi_1 \) or \( \phi_2 \) in the failure criterion.

Considering next the \( \phi \) value for experiment C-2 with coquina, i.e. 29.97°, this value is close to the value \( \phi = 30.12° \) for the C-1 experiment. The small difference could be attributed to the difference in the sample size (30 in C-1 and 100 in C-2). The same conclusion can be drawn relative to the coefficient of variation, \( s_\phi \).

A comparison of \( \phi \) values from experiments CAI and CAII shows that, in spite of the larger size of the CAII shells relative to CAI shells, \( \phi \) in the latter case was slightly lower. Nevertheless, based on the \( s_\phi \) values, it is noted that \( \phi \) for CAII showed a greater spread about \( \phi \) than for CAI. Considering, for example, \( \phi = \phi \pm s_\phi \) for CAI and CAII, the corresponding ranges of \( \phi \) would be 27.75° to 30.67° and 25.63° to 31.17°, respectively. This shows that the CAII shells generally attained both lower as well as higher \( \phi \) values.

The O experiment showed that \( \phi \) for oyster was much higher than those for the other shells. A comparison between \( \phi \) values for oyster (38.50°) and coquina (29.97°) reveals that the former was
considerably larger than the latter (by 28°), in spite of the fact that the $\beta$ values were not significantly different (0.49 for coquina and 0.44 for oyster). The particle size, represented by $d_s$, also showed a similar increase (9 mm for coquina and 41 mm for oyster). These trends in $\phi$ and $d_s$ suggest a positive correlation between these two parameters for coquina and oyster. However, this observation is not corroborated by the results from CAI and CAII experiments. In this case, even though $d_s$ increased from 13 mm to 16 mm from CAI to CAII (and $\beta$ remained invariant, i.e. 0.57), $\phi$ decreased, as noted, from 29.21° to 28.40°. These observations point to the importance of surface roughness defined by shell sculpture in the determination of $\phi$. Indeed, the uneven convex surface of oyster shells probably resulted in a greater degree of interlocking between these shells than in the case of coquina. The $\phi$ value could thus have been expected to be

### Table 4. Values of W and $\alpha$ for various functions of $\phi$, $\phi_1$ and $\phi_2$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Variable</th>
<th>W</th>
<th>$Pr &lt; W$ (or $\alpha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-1</td>
<td>$\phi$</td>
<td>0.910</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>$\log\phi$</td>
<td>0.915</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>$\cot\phi$</td>
<td>0.919</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>$\log(\cot\phi)$</td>
<td>0.913</td>
<td>0.021</td>
</tr>
<tr>
<td>C-1</td>
<td>$\phi_1$</td>
<td>0.969</td>
<td>0.543</td>
</tr>
<tr>
<td></td>
<td>$\log\phi_1$</td>
<td>0.970</td>
<td>0.564</td>
</tr>
<tr>
<td></td>
<td>$\cot\phi_1$</td>
<td>0.970</td>
<td>0.563</td>
</tr>
<tr>
<td></td>
<td>$\log(\cot\phi_1)$</td>
<td>0.969</td>
<td>0.553</td>
</tr>
<tr>
<td>C-1</td>
<td>$\phi_2$</td>
<td>0.911</td>
<td>0.018</td>
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<tr>
<td></td>
<td>$\log\phi_2$</td>
<td>0.913</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>$\cot\phi_2$</td>
<td>0.915</td>
<td>0.025</td>
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<tr>
<td></td>
<td>$\log(\cot\phi_2)$</td>
<td>0.911</td>
<td>0.019</td>
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<td>O</td>
<td>$\phi$</td>
<td>0.936</td>
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<tr>
<td></td>
<td>$\log(\cot\phi)$</td>
<td>0.939</td>
<td>0.110</td>
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### Table 5. Values of $D$ and $\alpha$ for various functions of $\phi$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Variable</th>
<th>D</th>
<th>$Pr &gt; D$ (or $\alpha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-2</td>
<td>$\phi$</td>
<td>0.094</td>
<td>0.029</td>
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<tr>
<td></td>
<td>$\log\phi$</td>
<td>0.087</td>
<td>0.061</td>
</tr>
<tr>
<td></td>
<td>$\cot\phi$</td>
<td>0.081</td>
<td>0.102</td>
</tr>
<tr>
<td></td>
<td>$\log(\cot\phi)$</td>
<td>0.090</td>
<td>0.045</td>
</tr>
<tr>
<td>CAI</td>
<td>$\phi$</td>
<td>0.191</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>$\log\phi$</td>
<td>0.202</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>$\cot\phi$</td>
<td>0.209</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>$\log(\cot\phi)$</td>
<td>0.197</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>CAII</td>
<td>$\phi$</td>
<td>0.122</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>$\log\phi$</td>
<td>0.137</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>$\cot\phi$</td>
<td>0.151</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>$\log(\cot\phi)$</td>
<td>0.132</td>
<td>&lt;0.01</td>
</tr>
</tbody>
</table>

### Table 6. Mean and coefficient of variation of $\phi$, $\phi_1$ and $\phi_2$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Variable</th>
<th>Mean (deg.)</th>
<th>Coefficient of Variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-1</td>
<td>$\phi$</td>
<td>30.12</td>
<td>4.56</td>
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<td></td>
<td>$\phi_1$</td>
<td>35.73</td>
<td>3.32</td>
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<td></td>
<td>$\phi_2$</td>
<td>39.31</td>
<td>3.03</td>
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<tr>
<td>C-2</td>
<td>$\phi$</td>
<td>29.97</td>
<td>4.48</td>
</tr>
<tr>
<td>CAI</td>
<td>$\phi$</td>
<td>29.21</td>
<td>5.00</td>
</tr>
<tr>
<td>CAII</td>
<td>$\phi$</td>
<td>28.40</td>
<td>9.74</td>
</tr>
<tr>
<td>O</td>
<td>$\phi$</td>
<td>38.60</td>
<td>5.23</td>
</tr>
</tbody>
</table>

Figure 4. Cumulative frequency distribution of $\cot\phi$ for coquina valves. Straight line corresponds to normal (Gaussian) distribution.
higher for oyster than for coquina, even if the two species had the same size.

As noted earlier, the natural slope of the oyster bank at Marineland (Figure 2) was in the range of 40°-45°, maximum. These were not precise measurements, but it is interesting to note that φ in O experiment was 38.6°, which is somewhat lower than 40°-45°. This would be expected because the bank or pile angle would be closer to φ₂, which is larger than φ.

From the above observations it is evident that it is essential to consider roughness in quantitative terms in any attempt to correlate φ with the geometric parameters for shells. For individual grains sliding over a bed with fixed roughness, the roughness height was shown by MILLER and BYRNE (1966) to be a useful descriptor of the influence of grain roughness on φ. However, there appears to be no similar, easily determinable parameter useful for a loose bed of shells. It is, therefore, necessary to define roughness by identifying the species.

CONCLUSIONS

The angle of repose, φ, can be determined from laboratory experiments in which the bed is tilted slowly until incipient motion is observed. This angle is required for calculating the critical shear stress for bed erosion. Further tilting of the bed can yield additional angles of stability such as φ₁ and φ₂, both of which are numerically greater than φ. However, the use of φ₁ or φ₂ in place of φ for estimating the critical shear stress for erosion will result in a larger than desired value of the critical shear stress.

Experimental determination of φ values for beds of coquina, two mixtures of chione and ark and oyster shells yielded mean values ranging from 28.40° to 38.60°. An examination of the distributions of φ about the mean φ showed that neither φ nor log φ, cot φ, or log(cot φ) were normally distributed in the case of beds of coquina or of mixtures of chione and ark. Although log φ, cot φ, and log(cot φ) from experiments with oyster beds were found to be normally distributed, this conclusion is based on only 30 measurements. Further experiments with greater number of measurements (at least 100), would be required to support the hypothesis. In general it may be concluded that there is insufficient evidence to justify the consideration of the distribution of φ, or of log and cot functions of φ, to be normal. The non-normality of φ in all cases as well as that of the angle φ₂ for coquina is attributed to observer bias in reporting "extreme" angles based on visual criteria. The observed normal distribution of the angle φ₁ for coquina is likely to be due to the relative ease with which this angle can be measured.

There appears to be no easily quantifiable relationship between φ and parameters which characterize shell geometry and sculpture. When dealing with bivalve shells it is therefore recommended that the angle of repose data be specified in terms of: (1) the mean, φ̄; (2) the coefficient of variation, sφ; (3) the shell species (for specifying surface roughness); (4) the three mutually perpendicular axial dimensions, a, b, c; (5) the diameter (mean value), d₀; (6) the coefficient of variation, s d₀, and (7) the shape parameter (mean value), β. Alternatively, the last three may be defined by other available granulometric parameters.

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LITERATURE CITED


