Problems in Linking the Threshold Condition for the Transport of Cohesionless and Cohesive Sediment Grain

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ABSTRACT


The threshold condition for the incipient transport of sediment linking cohesionless to cohesive size ranges is revisited using known concepts. The functional dependence of the threshold or critical shear stress on the principal parameters characterizing the sediment bed is examined, while recognizing that the actual relationships among these parameters remain largely unknown. For marine muds, until further experimental evidence is obtained, it is argued that erosion shear stress formulations for cohesionless sediment transport should be restricted to sediments containing grains larger than about 20 μm, and empirical erosion shear strength formulations be applied to sediments smaller than this size. Given this limit, the classical definition of silt size range (2 μm to 62 μm) based on the plastic properties of soil does not appear to be very useful in delineating the domains of cohesion-dominated and cohesionless sediment transport.

ADDITIONAL INDEX WORDS: Erosion, marine mud, rheology.

INTRODUCTION

The development of the fundamentals of cohesionless sediment transport preceded those of cohesive sediment transport by several decades. The latter has followed a course that has been seemingly independent of the former, using a somewhat different conceptual framework. A consequence has been that in the fine-grained size range within which the effects of cohesion are manifested with decreasing grain size, the compatibility and the domains of validity of coarse and cohesive sediment transport formulas have not been examined carefully. The ensuing problem of application has been recognized for years, yet difficulties in deciding when to use a particular formula and under what conditions persist. The problem is compounded by the natural occurrence of mixtures having heterogeneous properties. For the present purposes, however, we will restrict the treatment to uniform (single grain-sized) sediments.

A review of the literature on sediment transport suggests that considerable additional research may be required to link cohesionless and cohesive sediment transport theories. We have selected a few past studies to explore the possibility of developing a practical insight into the linkage issue, and narrowed the objective to revisit the condition of incipient transport of grains at the bed surface. Specifically, we will consider the threshold condition for grains under unidirectional flows, as would be applicable to steady and quasi-steady, e.g., tide-driven, currents. Within that context we will attempt to highlight gaps in our ability to determine the threshold condition over the range of grain sizes from coarse to fine.

INCIPIENT TRANSPORT

Consider the forces acting on a grain at the surface of a horizontal bed subjected to unidirectional turbulent flow as shown in Figure 1. In the cohesive size range the material will be considered to be flocculated. The physics of floc transport is complicated by the interaction between flow and floc properties. For instance, flocs that are partially exposed at the bed surface tend to deform elastically under shear flow, which consequently leads to a reduction in the interfacial drag (GUST, 1976). In order to generalize the present development over a wide range of grain sizes, we will assume the grain to be non-deformable, and to possess a physically recognizable identity either at rest or in motion. The ensuing limitations in describing the relevant physics notwithstanding, these assumptions will enable us to include cohesive materials in this approximate treatment.
In what follows it will suffice to consider the problem as one involving turbulence-mean flow-dependent quantities, without explicit reference to the essentially stochastic nature of the fluctuating quantities. Thus we will replace instantaneous forces by their corresponding time-mean values. This consideration is partly supported by the observation that flocs at the surface of soft, cohesive soil beds can begin to be entrained at flow conditions characterized by the presence of a viscous sublayer that is much thicker than the floc diameter (Mehta, 1991a). Thus, at least under this situation, the occurrence of turbulence in the body of flow need not be invoked formally as a condition for the analysis of incipient transport.

In a cohesive bed each grain is held by cohesive forces which bond the grain to its neighbors at discrete points of contact. Grain-grain contact can thus be broken by shear or normal forces. Clay cohesion, which arises from electrochemical effects, is often significantly modulated in marine muds by biochemical factors (e.g., mucopolysaccharide binding). Here we will consider cohesion to be akin to electromagnetic attraction to the extent that for rupturing the inter-granular bond a normal tensile force (e.g., due to lift) is required as an agency opposing cohesive attraction. Shearing (due to drag) is opposed by resistance due to cohesion, friction and interlocking. The resistive effect of friction and interlocking, also present in lifting the grain, can be incorporated by way of a coefficient as a modifier of cohesion (e.g., Partanen, 1977). With reference to Figure 1 the hydrodynamic forces include the lift, L, and the drag, D, while the (active) cohesive force, Fc, which is additive to the buoyant weight W of the grain, will oppose the lift. Fr, is the reactive shear force, equal in magnitude and opposite in direction to D.

The spatial-mean bed plane passes through point a, which lies above the center of mass of the grain at point b (see e.g., Christensen, 1975). Consequently, while D, and hence Fr, pass through a, L and W act through b. Thus, since all the forces do not pass through a single point, an evaluation of the condition for grain motion must consider both the forces and the moments. The active moment of interest here is Dab, which can assist the grain to be entrained by causing it to roll into the fluid. For simplicity we will assume that D acts through point b, thus ignoring the typically small couple arm, ab.

The condition for incipient transport in terms of the active forces corresponding to Figure 1 is shown in Figure 2. This condition indicates that, in this physical state of the grain, the resultant of the drag force, D, and the net downward force, W + Fc - L, must subtend an angle \( \phi \), equal to the angle of repose. By definition, the resultant therefore passes through c, the point of intergranular contact. The angle of repose has a clear physical meaning for cohesionless sediments, and is mainly dependent on grain size. However, for cohesive sediments it becomes a coefficient that embodies shear resistance (Lambe and Whitman, 1969). In either case the effect of Dab can be incorporated in the magnitude of \( \phi \) without loss of generality.

From Figure 2 the well-known condition for incipient transport is:

\[
\tan \phi = \frac{D}{W + F_c - L}
\]  

Both D and L are consequences of the horizontal flow velocity u (and are proportional to \( u' \) in turbulent flow). A characteristic reference height for u is the top of the grain (Christensen, 1975).
Therefore, given $\tau_\theta = \text{threshold or critical bed shear stress}$, $d = \text{a representative grain size}$, $\alpha_1 = \text{an area shape factor}$, and $\alpha_2 = \alpha_1 C_\nu \alpha_2 C_{\text{m}}$, where $C_\nu$ and $C_{\text{m}}$ are the lift and drag coefficients, respectively, we have $D = \alpha_1 \tau_d d^2$, and $L = \alpha_2 \tau_d d^2$. The buoyant weight, $W = \alpha_1 \gamma (\rho_d - \rho)d^2$, where $\alpha_1$ is a volumetric shape factor, $\gamma = \text{acceleration due to gravity}$, $\rho_d = \text{grain (or floe) density}$ and $\rho = \text{fluid (water) density}$. Substituting for these quantities in Eq. 1, the following relationship is obtained:

$$\frac{\tau_\theta}{g(\rho_d - \rho)d} = \frac{\alpha_1 \tan \phi}{(\alpha_1 + \alpha_2 \tan \phi)} + \frac{F'_\nu \tan \phi/(\alpha_1 + \alpha_2 \tan \phi)}{g(\rho_d - \rho)d^2}$$

(2)

where the term on the left hand side is recognized as the Shields' parameter, $\psi$, representing the ratio of drag force to buoyancy. The first term on the right hand side, referred to as $\theta$, is seen to be a sediment-dependent parameter (see discussion later), and the second term is essentially the ratio of cohesive force to buoyancy.

When cohesion is important, Eq. 2 has similarities with the well-known Coulomb's equation, $\tau = \sigma \tan \phi + C$, where $\tau$ is the soil shear strength, $\sigma$ the effective normal stress, and $C$ a measure of cohesion (LAMBE and WHITMAN, 1969). In fact, Eq. 2 can be thought of as a form of Coulomb's equation applicable to the bed surface, where there is no effective stress.

When cohesion is unimportant (i.e., $F'_\nu$ is negligible), the second term on the right hand side is essentially zero. In that case Eq. 2 becomes a representation of Shields' entrainment relationship for coarse sediment, $\psi = \psi(\theta)$, with $\tau_\theta$ commonly denoted by the symbol $\tau_c$. In general, $\theta$ is a function of the roughness Reynolds number, $R_c = u^* d/\nu$, where $u^* = (\tau_c/\rho)^{1/2}$ is the critical friction velocity and $\nu = \text{kinematic viscosity of water}$. $R_c$ can be viewed as the grain size, $d$, normalized by $u^*/\nu$, where the latter is proportional to the viscous boundary layer thickness. Note that $R_c$ is more generally defined as $u^* k_\nu /r$, where $k_\nu = \text{Nikuradse's equivalent bed roughness}$ (GRAF, 1971). Here, however, we will conveniently assume the relative grain size, $d/k_\nu$, to be equal to one for defining $R_c$. For $R_c < 5$, the boundary layer is thicker than $d$, and viscous stresses entrain the grain. For $R_c > 100$ the viscous effects are negligible, and in this fully rough turbulent flow $\theta$ is independent of $R_c$. Equation 2, under the stated assumptions, actually applies only to the last case, since $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\phi$ are constants for a given sediment bed, hence $\theta$ in this case does not depend on $R_c$. It is, however, possible to theoretically extend the applicability of Eq. 2 to the transition range between viscous and fully rough turbulent behaviors, i.e., $5 < R_c < 100$, by accounting for the dependence of the velocity, $u$, on $R_c$ (CHRISTENSEN, 1975).

Criteria that are analogous to Eq. 2 have been proposed previously using both force and momentum balance approaches. Examples using forces include SUNDBORG (1956), GRAF (1971), PARTHENIADIES (1977), CHIEN and WAN (1986), CHU and WANG (1989) and DADE et al. (1992). MIRTSKHOULOVA (1991) has derived a criterion using momentum balance. In it, when it is assumed that all the moment arms are proportional to the grain diameter, the form of Eq. 2 results.

**CRITERIA FOR COHESIONLESS AND COHESIVE BEDS**

It is evident from the terms present in Eq. 2 that Shields' relationship is not applicable to situations involving flocculated sediments, and there is the further implication that extrapolation of this relationship into the cohesive size range cannot be achieved unequivocally without including some measure of cohesion as a behavior-governing parameter. Such extrapolated relationships are commonly reported in literature on sediment transport in engineering and marine geology. For example, Figure 3 from VANONI (1975) presents a Shields-type relationship compiled by HJULSTRÖM (1935) using data from prior laboratory studies relating the critical velocity at threshold transport to grain size. A relationship between the critical velocity and the corresponding grain diameter is inherently approximate. This is so because it can be easily shown that such a relationship cannot be independent of water depth. Notwithstanding this limitation however, Figure 3 shows this relationship to be applicable down to a size of 1 $\mu$m, the typical dispersed clay diameter. The data for sediment sizes less than 100 $\mu$m were taken from an extensive field data compilation by FORTIR and SCOBIE (1926). VANONI (1975) recognizes the significance of cohesion for fine-grained materials; yet, in contradiction to the meaning of Eq. 2, cohesion is not represented as an independent factor in Figure 3. Hence an unambiguous value of the critical velocity (and stress therefrom) for a cohesive soil bed cannot be obtained from this plot. Indeed, in a highly systematic laboratory study
on the erosion of clays in water of variable pore water ions and their concentrations, KANDIAH (1974) showed that for the same clay, changing the pore water chemical composition or pH can lead to threshold stresses that easily differ by an order of magnitude.

A second difficulty of equal significance is the fact that the floc diameter is typically quite weakly correlated to the dispersed grain diameter, hence the latter is not a good measure of the floc size. Finally, in analogy with Shields' relationship, which is re-plotted as a mean trend line in Figure 3 based on the data of SHIELDS (1936), small values of d would tend to suggest that the incipient entrainment of flocs occurs solely under viscous-dominated conditions, which is not always the case. In fact, soil erosion data (e.g., TASK COMMITTEE of ASCE, 1968) indicate that some resilient clay beds do not erode until \( R_e \) attains very high values, on the order of several hundred.

In the cohesive size range, by: (1) selecting \( \tau_c = F_r \tan \phi / (\alpha_c + \alpha_r \tan \phi) d^2 \) in Eq. 2, where \( \tau_c \) is defined as the cohesive bed shear stress with respect to erosion, and (2) ignoring the first term on the right hand side (or, recognizing that in this case \( \alpha_c, \alpha_r, \alpha_r, \) and \( \phi \) are cohesion-dependent coefficients, and thus effectively absorbing the first term on the right hand side into the definition of \( \tau_c \)), Eq. 2 reduces to the condition, \( \tau_c = \tau_e \). This is the threshold condition for the erosion of cohesive soil bed surfaces, i.e., erosion commences when the applied ("critical") bed shear stress equals the erosion shear strength, as demonstrated by, among others, PARCHURE and MEHTA (1985). However, \( F_r \) or \( \phi \) cannot be estimated easily for an eroding cohesive bed. Thus, \( \tau_e \) must be determined experimentally for specific cohesive materials.

A recent evaluation of several previous laboratory studies (MEHTA, 1991b) suggests that \( \tau_e \) reasonably correlates with the solids volume fraction \( \phi_s \). In general

\[
\tau_e = \alpha_c (\phi_u - \phi_m) \phi_s^\beta
\]

where \( \phi_m \) is the critical solids volume fraction such that for all \( \phi_u < \phi_m \) the soil does not possess a measurable structural integrity (even though it does have cohesion), and is thus not a bed but is essentially a fluid-supported slurry. The coefficients \( \alpha_c, \beta \) depend on sediment-specific cohesive properties characterized by clay composition, cation exchange capacity, fluid salinity, temperature and so on. Note that \( \phi_u = 1 - n \), where \( n \) is porosity. Since \( n \) typically decreases with depth in a cohesive bed and may also change with time due to consolidation, the spatial and

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Figure 3. Critical velocity as a function of grain size from data reported by Hjulström (1935) from other experimental sources, and the work of Shields (1936) for quartz sand. The three curves of Hjulström correspond to mean, the upper and the lower bounds of his data. Adapted from Vanoni (1975).
temporal variability of $\tau$, must be appropriately accounted for when calculating bed erosion (Par
chure and Mehta, 1985).

Rheological studies (by, e.g., James et al., 1988) have established that $\phi_w$ is the value of $\phi_a$ above which the occurrence of a space-filling granular network leads to a rapid increase in bed rigidity as defined by the shear modulus of elasticity. Typically, $\phi_w$ is small in comparison with $\phi_a$, on the order of 0.01, so that it is permissible to ignore it in studies on beds having relatively high values of $\phi$. An example of the applicability of Eq. 3 ignoring $\phi_w$ is shown in Figure 4 for an estuarine mud (Kusuda et al., 1984). Given $\tau$, in Pa, values of $\alpha = 6.5$ (Pa) and $\beta = 1.6$ yield the best fit equation. The $\beta$ value is fairly typical, but $\alpha$ can be as much as an order of magnitude higher, depending upon the sediment composition (Mehta, 1991b). In fact $\alpha$ is found to be non-linearly dependent on the grain size (Buscall et al., 1988; Dade and Nowell, 1991).

**EROSION SHEAR STRESS AND STRENGTH**

From what we have stated thus far it is recognized that: (1) for cohesionless soils the threshold condition for transport is specified by a particular value of the applied stress, $\tau$, that causes the grain on the bed surface to be entrained, and (2) the threshold condition for a cohesive soil bed occurs when the applied stress equals the shear strength, $\tau$, associated with the bed surface flocs. In the latter case the floc is detached from its neighbors, to which it is otherwise linked by cohesion, and is entrained. Note also that the “grain” density, $\rho_v$, is typically on the order of 1,010 to 1,020 kg/m$^3$, which is considerably smaller than 2,650 kg/m$^3$ for quartz sand for example. Thus, flocs typically have substantially lower buoyant weights than cohesionless materials. In turn, once the cohesive bonds are broken, the floc is readily brought into suspension.

Based upon the preceding discussion, we may consider the possibility of proposing the following parametric dependence of the threshold stress, $\tau$, in the functional form:

$$f(\tau, \rho_w, \rho, d, g, \phi_a(n), F) = 0$$  (4)

and examine its behavior. To render Eq. 4 amenable to a coherent interpretation and following, among others, Valembois (1960), we will eliminate $u^*$ by combining $\psi$ and $R_w$, to yield a dimensionless grain diameter, $d = d/[\rho^2 g(\rho_v - \rho)]^{1/3}$. In this ratio the denominator is a characteristic diameter, and $d$ qualitatively represents the ratio of buoyancy to viscous dissipation. Selecting $\rho_v = 2,650$ kg/m$^3$ and $\rho = 10^3$ m$^2$/s for water, the characteristic diameter becomes 0.04 mm (40 $\mu$m), which is in the silt size range, and thus offers a convenient means for normalizing $d$. We may consequently rewrite Eq. 4 as

$$\psi = \psi(d, \phi_a(n), F)$$  (5)

For coarse sediment, e.g., $d \geq 62 \mu$m, $\tau \approx \tau_s$, and Eq. 5 can be expressed in the form of Shields’ relationship, $\psi = \psi(d)$, as shown by Ackers (1972). For a cohesive material, e.g., $d \leq 2 \mu$m, $\tau \approx \tau_s = f(\phi_a, F)$, which is the functional form of Eq. 3.

**THE TRANSITION RANGE**

In the transition range from 62 $\mu$m to 2 $\mu$m, conventionally considered to constitute the entire silt size range and defined as such on the basis of the plastic properties of soil (Lambe and Whitman, 1969), the behavior of the parameters in Eq. 4 is generally not well-known. The work of Dade et al. (1992) is a noteworthy exception. These investigators have provided a theoretical basis relating $\tau_s$ to the yield stress, $\tau_y$, over 1 to 25 $\mu$m size range. Since for a given sediment $\tau_y$ is strongly

![Figure 4. Erosion shear strength versus solids volume fraction for an estuarine mud (after Kusuda et al., 1984).](image-url)
correlated to $\phi_s$ (Mehta, 1991b), Eq. 5 holds in this case as well.

Bagdold (1966) among others suggested that his theoretical formulations for coarse grain suspended load transport rate would be valid down to $d = 15 \mu m$. Mantz (1977) confirmed this limit using crushed silica, a “clean” sediment, and extended Shields’ relationship accordingly, as have Wiberg and Smith (1987). This observation leads to the inference that with decreasing grain size the effect of cohesion perhaps becomes significant rather abruptly. In other words, the parameters in Eq. 4 do not vary gradually over the silt size range, but relatively rapidly over a comparatively vary narrow range, which may possibly be represented by a single size for practical purposes. Experimental evidence obtained by, among others, Migniot (1968), in terms of quiescent settling of a large number of flocculated marine muds in water of 30 ppt salinity in laboratory columns, seemingly corroborates the occurrence of such an abrupt change.

Using Migniot’s data relating floc settling velocity with the corresponding Stokes settling velocity of the constituent particles, the following relationship between the floc diameter, $d_f$, and the median particle diameter of the dispersed sediment, $d_{50}$, can be obtained (Mehta and Lott, 1987):

$$d_f = K \left( \frac{\rho_s - \rho}{\rho_1 - \rho} \right)^{1/2} d_{50}^{n+1} \quad (6)$$

Here $\rho_s$ is the dispersed sediment grain density, $\rho$ is the floc density and the coefficient $K = 15.8$, when the diameters are specified in $\mu m$. The limiting diameter below which $\rho_s$ begins to decrease relative to $\rho$, can be obtained by setting $(\rho_s - \rho)/(\rho_1 - \rho) = 1$, which yields $\rho_s = 21 \mu m$. By examining the settling of a marine mud in tap water under flow, which did not promote significant floculation, Dixit et al. (1982) found the limiting diameter to be 10 $\mu m$.

Mehta et al. (1989) combined the settling data of Migniot (1968) and Chase (1979) to yield the results shown in Table 1. Note the effects of decreasing primary (dispersed) grain diameter, $d$, on the Stokes settling velocity of the primary particle, the floc settling velocity, the floc diameter and the ratio of floc to Stokes velocities. While Stokes velocity expectedly decreases quite rapidly as $d$ decreases from 20 to 0.2 $\mu m$, the floc velocity and diameter hardly change, in qualitative agreement with Eq. 6. Consequently, the velocity ratio, which is close to one at 20 $\mu m$, increases to 4,600 at 0.2 $\mu m$. This increase is an indirect but very reliable indicator of the effect of cohesion, which is seen to be very small at 20 $\mu m$.

The preceding observations suggest that until further experimental evidence is gathered, the 20 $\mu m$ size may be considered practically to be the dividing size differentiating cohesive and cohesionless sediment transport behaviors of marine sediments with respect to the threshold condition. It is noteworthy that Ackers and White (1973) considered the dividing size to be $d_f = 1$, i.e., $d = 40 \mu m$. This value is somewhat higher but is consistent with the fact that, at least in some cases, especially ones involving material that is heterogeneous in terms of size and mineral composition, cohesion plays a role in governing the behavior of sediment larger than 20 $\mu m$. However, for practical applications this effect is not overly significant until the size reduces to around 20 $\mu m$, when cohesion becomes important relative to buoyancy. It is noteworthy that, based on a different approach to the problem involving a careful chemical analysis of glaciomarine sediments, Stevens (1991) has proposed the 16 $\mu m$ boundary between sediments that flocculate significantly, and coarser silts, which is close to 20 $\mu m$. Finally we note again that wholly non-clay minerals as small as 10 $\mu m$ tend to exhibit cohesionless behavior. However, these are special cases, not representative of typical muds.

### CONCLUDING COMMENTS

As noted the classical definition of the silt size range (62 $\mu m$ to 2 $\mu m$) is based on the degree of plasticity possessed by the soil. While this size-based definition has proven to be very useful in soil mechanics and geotechnical engineering, there appears to be no cogent basis to retain its use in delineating the domains of water-borne cohesive and cohesionless sediment transport. Indices based on plastic properties do not uniquely correlate...
with, for instance, bed surface erosion of cohesive soils (Partheniades, 1965), and cohesion is not highly important for soils with particles larger than 20-40 μm. For purposes of sediment transport, the settling velocity is unquestionably a fundamental property that takes precedence over grain size. However, if we must continue to use the grain size as a transport-characterizing parameter for obvious convenience, then the basis for using the conventional sand/silt/clay size classification for transport must be revised after a careful examination of transport in the “silt” size range. It is conceivable that further examination of the problem of transition between cohesionless and cohesive behaviors will require a careful evaluation of the rheology of bottom sediments in terms of their elastic and dissipative properties. Such an evaluation should assist in elucidating the nature of Eq. 4 in the transition size range.

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