A Hydrodynamic Model for the Río de La Plata, Argentina

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ABSTRACT


As part of a long term development plan for the port of Montevideo, Uruguay, a finite element mathematical model was constructed to characterize the global hydrodynamic regime of the Río de la Plata. The model takes into account the main factors that determine the hydrodynamic regime of the river; i.e., fluvial flow rates, tides and winds, and provides the necessary boundary conditions for further analyses of any particular limited area within the river. In its most general version, the model admits any law regarding water levels and velocities at the boundaries, and allows for variable bottom roughness, the Coriolis effect, winds and atmospheric pressures, salinity gradients and variable boundaries. It was calibrated with 1-day water level records from seven tide stations, and 7-day hourly wind records from nine stations along the river. A detailed analysis of the influence of wind action upon the Río de la Plata was also performed.

ADDITIONAL INDEX WORDS: Río de La Plata, mathematical model, finite elements.

INTRODUCTION

Flowing between Argentina and Uruguay, the Río de La Plata is formed by the confluence of the rivers Paraná and Uruguay (Figure 1). With a length of approximately 320 km, a width varying from 35 km (La Plata-Colonia) to 220 km (Punta Rasa-Punta del Este), and a total area of about 30,212 km², its mean depth is, however, only 10 meters. The sediment transported in huge quantities by the rivers Paraná and Uruguay is responsible for the existence of several sand banks that constitute one of its main features. Because of the downstream progress of the sediment, navigation in the Río de la Plata would be impossible were it not for a set of dredged channels that enable large ships to traverse it.

For modelling purposes, the Río de La Plata can be divided into three main regions or areas, each having its own characteristics. The inner region extends from the confluence of the rivers Paraná and Uruguay to an imaginary straight line joining Buenos Aires to Colonia. The Paraná delta and the Playa Honda shoals are the chief morphological features of this area. The intermediate region goes from the above-mentioned line to another imaginary line connecting Punta Piedras with Montevideo. This area is characterized by the Banco Ortiz, a very large sand bank already marked in a chart of 1875, and that seems to have remained unchanged ever since. Its dimensions reveal its importance: 100 km long, 40 km wide, and 3 m deep on the average. Finally, the outer region extends from the Punta Piedras-Montevideo line to the mouth of the river, taken conventionally as a straight line joining Punta Rasa to Punta del Este. This region can be divided further into four well distinct zones: Canal Punta Indio, with a depth increasing downstream from 13 to 30 m; Bahía Samborombom, a 2 to 5 m deep shoal; Banco Ingles and Banco Arquimides, whose very stable depths are less than 1 m; and the middle zone with Banco Rouen dipping gently towards the continental shelf.

FLUVIAL ACTION

The rivers Paraná (20,000 m³/sec mean flow rate) and Uruguay (6,000 m³/sec mean flow rate) have different flow regimes. According to some available records (DNCPYVN, 1969), the river Uruguay has its largest flow rate during the period...
May–October (southern hemisphere winter season), and its smallest flow rate during the period January–March (southern hemisphere summer season). The river Paraná, on the other hand, (from 1904–1930 and 1954–1959 statistics) shows its largest and smallest flow rates during the periods March–May (southern hemisphere autumn season) and September–October (beginning of the southern hemisphere spring season), respectively. There is also a network of small rivers on the Uruguayan side that contribute with a total flow rate of about 700 m³/sec, which is of minor importance.

**METEOROLOGY**

The climate of the central southern region of Argentina and of the Rio de La Plata has its origin in the Pacific Polar Front which is formed through the combination of subtropical air masses arising from the huge, semipermanent Pacific anticyclone and polar air masses originating in the Antarctic region. The barometric lows that frequently cross the territory of Argentina arise mostly from this front.

Of more importance from the viewpoint of weather conditions in the Rio de La Plata are the
anticyclonic cells that separate from the Pacific anticyclone and travel towards the ENE affecting areas of considerable extent. These anticyclonic cells are always associated with polar fronts coming from the south. The range, duration and direction of these fronts are closely related to the path, intensity and speed of the anticyclonic cells. If the center of the anticyclonic cell travels across the Argentine territory at a relatively high latitude (about 45°S), the polar front approaches the Rio de La Plata from the southeastern sector, giving rise to "southeasterlies". As the anticyclone continues its path towards the ENE, winds shift from SE to NE. If, on the other hand, the path of the anticyclonic cell is located at an intermediate latitude (about 35°S), the polar front comes from the SW producing "line squalls". Winds constitute a very important driving force in the Rio de La Plata, for they affect the water level and the currents in a significant way.

TIDE

The tide in the Río de La Plata is considered to be the combined effect of two tidal waves. One of them enters the river from the north along the Uruguayan coast, and the other from the south along the Argentine coast; the latter being the stronger (BALAY, 1961). On reaching the outer boundary of the Río de La Plata, the resultant tidal wave has mean ranges of 1.10 m at Punta Rasa and 0.25 m at Punta del Este (SHN, 1991). Since it takes this wave about 12 hours to travel through the river, two high waters and one low water (or vice versa) take place simultaneously with very marked diurnal inequalities. Once within the Río de La Plata, the tidal wave is subjected to distortion, damping and amplification processes due to the difference between the epochs of the constituents of the two primary waves, the Coriolis effect in the outer region, and the progressive narrowing of the river. Additional damping comes from fluvial action and the presence of the Playa Honda shoals.

Ebb tide would seem to be the modelling factor of the bathymetry. This can be seen to the west of Barra del Indio, where the erosive processes have not only made a westward slope but are also responsible for the disappearance of Banco Curassier and Banco Gaviota which existed until 1967 (PARKER et al., 1985). A similar situation can be found at Costa Pavón, Uruguay.

CURRENTS

The available current measurements in the Río de La Plata cover a period of about 25 years. Since they have been intended mainly for navigation safety, the time series are of different length. Published data (LANFREDI et al., 1979; MAZIO, 1987) show a hydrodynamic zoning whose limits are given by the direction and speed of the currents. A net difference of about a tidal cycle can be observed in the current velocity (Table 1).

Although the presence of Banco Ortiz constitutes a natural barrier to the flow of the Paraná river, this does not prevent the Paraná from contributing to the residual current with a significant ebb current component.

Shear stresses from southwestern winds blowing over the Río de La Plata produce a cross circulation pattern from the Argentine shore to the Uruguayan shore.

MATHEMATICAL MODEL

General Considerations

Coastal works whatever they may be require as a general rule a reasonable knowledge of the wave climate, circulation pattern and sediment transport rate of the area where they are going to be set. Very often, however, practical and economical reasons restrict the collection of data to a limited number of points. Because of this, it becomes necessary to resort to mathematical models which, starting from a small number of field measurements, enable the designer to obtain a continuous description of the whole area under consideration from the viewpoint of its hydrodynamic regime.

As they now stand, two-dimensional, vertically integrated mathematical models when calibrated and applied adequately, are best suited to study drainage in shallow water coastal zones with total or partial mixing (FARRADAY et al., 1975; ROdrigues Vieira, 1984), as is the case of the Río de La Plata. Not only do these kinds of models allow one to characterize the circulation pattern of large domains in a quick and correct way, they also provide the framework for later refinements upon those areas for which detailed analyses are to be attempted through the use of nested models.
The following are some of the general criteria that should be taken into account in the development of such mathematical models:

(1) The boundary conditions should be simple to define and associated with a dominant situation (e.g., oceanic or fluvial boundaries).

(2) Nested models should be for local zones whose hydrodynamic regimes come from the response of a global, surrounding system to different and interacting driving forces.

(3) Suitable records representative of a mean year have to be available for the chosen boundaries in order to define the typical driving forces and their associated periods. In this way, it will be almost unnecessary to make synoptic measurements for calibrating purposes.

(4) The boundaries should never be located where a coastal engineering work is going to be built; otherwise, measurements will be useless.

(5) Field data arising in the form of time series have to be subjected to Fourier analysis and sinusoidal regression to identify and separate the driving forces (e.g., tide and winds).

THE GLOBAL MODEL FOR THE RIO DE LA PLATA

The need for a mathematical model of the hydrodynamic regime of the Rio de La Plata arose as part of a long term development plan for the Port of Montevideo, Uruguay (Rodrigues Vieira, 1987). The mathematical model was developed as a system of two nested models: a global model for the whole Rio de La Plata, and a local one for the Port of Montevideo and its entrance area. The former was intended to characterize the typical hydrodynamic situations within the Rio de La Plata, and to provide the latter with the corresponding boundary conditions. Because of its general interest, it is the global model that will be discussed here.

Basically, the hydrodynamic model regime of the Rio de La Plata is determined by the tide and the outflows of the rivers Paraná and Uruguay. Wind action, however, modifies this basic regime considerably. All these factors act simultaneously along the river in such a complicated way that water level and currents at each point are determined not only by local conditions, but by the global response of the whole system as well. Under these circumstances, and allowing for the large extension of the region of interest, it becomes a very difficult task to define suitable boundary conditions within the Rio de La Plata through field measurements alone. This is why a set of two nested models had to be developed for characterizing the hydrodynamic regime of both the Rio de La Plata and the Port of Montevideo.

The Hydrodynamic Equations for Shallow Water

Very long period waves (tides and storm surges) coming from the continental shelf enter the Rio de La Plata and propagate into it. Since the river is a very shallow water environment, it seems reasonable to begin the analysis of the global model with a description of how the three-dimensional Reynolds equations and the continuity equation can be brought together and applied to incompressible, turbulent flow in shallow water. The following expression, written in terms of time-mean values of pressure and velocity, can be a suitable starting point.

\[
\rho \left( \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} \right) = \rho f_i - \frac{\partial p}{\partial x_i} + \mu \nabla U_i - \frac{\partial}{\partial x_j} \left( \frac{\partial U_i}{\partial x_j} \right) \]

where \( U_i \) is the velocity along the \( x_i \) direction, \( f_i \) the force per unit mass, \( \rho \) the fluid density, \( p \) the pressure, \( \mu \) the dynamic viscosity of the fluid, and \( U_i' \), \( U_j' \), the Reynolds stresses. In turbulent flow, the stresses due to viscosity are negligible when compared with the Reynolds stresses, so the \( \mu \nabla U_i \) term can be disregarded in Equation 1.

In estuaries and coastal lagoons, which are essentially shallow water environments, the vertical velocity component is almost nil, so vertical inertial forces are negligible when compared with gravity and pressure forces. Under these circumstances, the momentum equation in the vertical direction becomes

\[
\frac{1}{\rho} \frac{\partial p}{\partial x_i} + g = 0
\]

where \( g \) is the acceleration due to gravity. Equation 2 is equivalent to considering a hydrostatic pressure distribution. Starting from Equation 2, and assuming a mean density \( \rho_0 \) over the water column, one can obtain the following expression

\[
\frac{\partial p}{\partial x_i} = \frac{\partial p_0}{\partial x_i} + g \rho_0 \frac{\xi}{\partial x_i} + \xi \int_{-\infty}^{+\infty} \frac{\partial p}{\partial x_i} \, dx \quad i = 1, 2
\]

where \( p_0 \) is the atmospheric pressure and \( \xi \) the water level.

Equation 3 shows the influence of horizontal
density gradients upon pressure forces and explains any possible velocity inversion on the bottom during ebb tide as well as the lowering of the point of maximum velocity during flood tide. If there is a strong turbulence, this effect becomes less important because of the momentum exchange along the vertical direction. In such a case, the flow remains well-mixed with an almost uniform velocity profile. In other cases, however, density variations can happen over the water depth, which results in a partially mixed flow. On these conditions, the velocity profile can be described by distribution functions $\phi$ such as

$$u_i = U_i(1 + \phi_i(x_i, t))$$

(4)

where $U_i$ is the mean velocity over depth, and

$$\int_{-h}^{h} \phi(x_n, t) \, dx_n = 0$$

(5)

and

$$\alpha_{ij} = \frac{1}{h + \xi} \int_{-h}^{h} (1 + \phi_i(x_n, t))(1 + \phi_j(x_n, t)) \, dx_n$$

(6)

Thus, if the flow is partially mixed, it will be characterized by the mean velocity and density over depth, $U_i$ and $\rho_m$, and by the correlation coefficients $\alpha_{ij}$.

Equation 3 can be substituted into Equation 1 and the resultant expression integrated vertically allowing for Equations 1, 2 and 3, and keeping in mind that the horizontal momentum exchange is negligible on the surface and the bottom (TSENG, 1975), and that the external horizontal forces are due to the Coriolis effect. The final expression is

$$\frac{\partial}{\partial t} (HU) + \frac{\partial}{\partial x_i} (\alpha_{ij} H U_j)$$

$$= (-1)^i \int_{-h}^{h} H U_i \, dx_i - g H \frac{\partial \xi}{\partial x_i}$$

$$+ \frac{H}{\rho_m} \frac{\partial \rho_m}{\partial x_i} \frac{\partial H}{\partial x_i}$$

$$+ \left( \frac{\partial}{\partial x_n} \int_{-h}^{h} \tau_{ij} \, dx_n + \rho_m \frac{\partial}{\partial x_i} \frac{\partial \xi}{\partial x_n} \right)$$

(7)

where $H = n + \xi$ and $t = 2 \Omega \sin \varphi$, $\Omega$ being the angular velocity of the Earth (7.272 $\times$ 10 $^{-11}$/sec) and $\varphi$, the local latitude. The shear stresses $\tau_{ij}$ and $\tau_{ij}$ will be considered later.

It is particularly interesting to analyse the inertia terms. If the correlation coefficients $\alpha$ are considered as isotropic then

$$\frac{\partial}{\partial t} (HU) + \frac{\partial}{\partial x_i} (\alpha H U_j)$$

$$= (h + \xi) \frac{\partial U_j}{\partial t} + U_i \frac{\partial \xi}{\partial x_i}$$

$$+ (h + \xi) \alpha \left( U_j \frac{\partial U_i}{\partial x_i} + U_i \frac{\partial U_j}{\partial x_i} \right)$$

$$+ \alpha U_i U_j \frac{\partial \xi}{\partial x_i} + (h + \xi) U_i \frac{\partial \xi}{\partial x_i}$$

(8)

From the continuity equation, $\frac{\partial U_i}{\partial x_i} = 0$; also, the last term in Equation 8 may be considered as negligible, so the inertia forces per unit mass become

$$\frac{\partial U_i}{\partial t} + \alpha U_i \frac{\partial U_i}{\partial x_i} + \frac{U_i}{h + \xi} \frac{\partial \xi}{\partial x_i} + \frac{\alpha U_i}{h + \xi} \frac{\partial \xi}{\partial x_i}$$

(9)

The first two terms represent the local and convective accelerations respectively, considering the depth as constant; i.e., disregarding any possible time and space variation of the amplitude of the oscillations of the free surface with respect to the mean depth. The third and fourth terms are corrections that must be introduced into the accelerations if the above oscillations are to be allowed for. The third term, for instance, can be important when the tide range is comparable with the mean depth. The fourth term can be neglected.

The terms $\tau_{ij}$ and $\tau_{ij}$ represent the shear stress due to bottom friction and the wind stress on the surface respectively, and may be expressed through the following empirical relations

$$\tau_{ij} = \rho_s g \frac{U_i}{C^2}$$

(10)

$$\tau_{ij} = \rho_s K W \cos \theta$$

(11)

where $C$ is the Chezy coefficient (m/s), $K$ a dimensionless friction factor, $W$ the wind velocity (m/sec), $\rho_s$ the air density, and $\theta$ the wind direction.

The integral terms express the lateral exchange of momentum over the water depth associated with the Reynolds stresses. It is assumed that these stresses are proportional to the rate of an-
gular deformation (Boussinesq approximation), so
\[
\tau_{ij} = -u_i u_j' = \nu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)
\]  
(12)
where \( \nu \) are the coefficients of turbulent viscosity.

In practice, however, a mean viscosity coefficient is considered over the water depth, so that
\[
\int_{x_0}^{x_1} \tau_{ij} \, dx_j = \int_{x_0}^{x_1} \nu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \, dx_j
\]  
(13)

where \( \epsilon \) is the turbulent exchange coefficient.

Taking into account the preceding expressions, the governing equations for turbulent, incompressible flow in shallow water are given by
\[
\frac{\partial U_i}{\partial t} + \frac{U_i}{h + \xi} \frac{\partial U_i}{\partial t} + \alpha U_i \frac{\partial U_i}{\partial x_i} = \left( -1 \right)^i f U_{i-1} - \frac{g}{\rho_0} \frac{\partial \xi}{\partial x_i} - \frac{1}{\rho_0} \frac{\partial p_{\xi}}{\partial x_i} + \frac{\rho_0 K W^\cos \theta}{\rho_0} \frac{\partial^2 U_i}{\partial x_i^2} + \frac{\epsilon}{\rho_0} \frac{\partial^2 U_i}{\partial x_i^2} - \frac{g U_i (U_i')^2}{(h + \xi) C_i^2} + \frac{\epsilon}{\rho_0} \frac{\partial U_i}{\partial x_i}
\]  
(14)

If the flow is well-mixed, the correlation coefficients \( \alpha \) are taken as unity.

\[\int_{x_0}^{x_1} \epsilon W_i \, d\xi = 0 \]  
(15)

where \( \epsilon \) represents the approximation errors and \( W \) the weighting functions and superscript \( e \) an element property. If the weighting functions are considered as variation of the approximation functions (Galerkin's method), one obtains, for instance
\[\int_{x_0}^{x_1} \epsilon W_i \, d\xi = 0 \]  
(16)

Because of the presence of non-linear terms (convection and bottom roughness), it is necessary to use an iterative procedure until convergence is attained. Unfortunately, these techniques are very time consuming. But there is a way out of this problem. Since the flow under study is a slow varying one, it is possible to consider a mean, steady-state equivalent flow at each carefully chosen calculation step. Thus, the initial conditions at each step are given by the final conditions at the previous one, thus avoiding the need for using any iterative method, though retaining the non-linearity of the above-mentioned terms along the whole calculation stage. For instance, Equation 14 becomes
\[
\frac{\partial U_i}{\partial t} + \frac{U_i}{h + \xi} \frac{\partial U_i}{\partial t} + \alpha U_i \frac{\partial U_i}{\partial x_i} = \left( -1 \right)^i f U_{i-1} - \frac{g}{\rho_0} \frac{\partial \xi}{\partial x_i} - \frac{1}{\rho_0} \frac{\partial p_{\xi}}{\partial x_i} + \frac{\rho_0 K W^\cos \theta}{\rho_0} \frac{\partial^2 U_i}{\partial x_i^2} + \frac{\epsilon}{\rho_0} \frac{\partial^2 U_i}{\partial x_i^2} - \frac{g U_i (U_i')^2}{(h + \xi) C_i^2} + \frac{\epsilon}{\rho_0} \frac{\partial U_i}{\partial x_i}
\]  
(17)

where \( U_i \) and \( \xi \) represent the initial conditions at each calculation step.

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Computational Procedure

The numerical solution of Equations 14 and 15 for the Rio de La Plata is based on the method of finite elements. The spatial discretization of the differential equations is obtained by using linear triangular finite elements with a formulation by weighted residues for the approximation in each sub-domain or element (Zienkiewicz, 1971). Figure 2 shows the grid of finite elements with its 716 nodes.

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\]  
(16)

where \( U_i \) and \( \xi \) represent the initial conditions at each calculation step.
Figure 2. Finite element grid for Rio de La Plata.

\[ + \int_{\Delta x} \frac{\hat{U}}{h + \xi} N_n \frac{\partial N}{\partial t} N_j \delta U_j \, d\Delta x \]
\[ + \int_{\Delta x} \left( \hat{U} \frac{\partial N}{\partial x} U_i + \hat{V} \frac{\partial N}{\partial y} U_j \right) N_j \delta U_j \, d\Delta x \]
\[ + \int_{\Delta x} g \frac{\partial N}{\partial x} \xi N_i \delta U_i \, d\Delta x \]
\[ - \int_{\Delta x} f N_i V_j N_j \delta U_j \, d\Delta x \]
\[ + \int_{\Delta x} \frac{1}{\rho_m} \frac{\partial N}{\partial x} N_i \delta U_i \, d\Delta x \]
\[ + \int_{\Delta x} g \frac{\partial N}{\partial x} \frac{h + \xi}{2} N_i \delta U_i \, d\Delta x \]
\[ + \int_{\Delta x} g N_i U_i \left( \hat{U}^2 + \hat{V}^2 \right) \frac{N_j \delta U_j}{(h + \xi)C^2} \, d\Delta x \]
\[ + \int_{\Delta x} \frac{KW^2 \cos \theta}{\rho_s \rho_n (h + \xi)} N_i \delta U_i \, d\Delta x \]
\[ - \int_{\Delta x} \frac{\epsilon}{\rho_m} \left( \frac{\partial^2 N_i}{\partial x^2} U_i + \frac{\partial^2 N_i}{\partial y^2} U_j \right) N_j \delta U_j \, d\Delta x \]
\[ = 0 \] (19)

This method has been used successfully by Taylor et al. (1973), among the others to solve two-dimensional flow in wide shallow estuaries.

Similarly, after simplifying and rearranging, the following linear system is attained for the rest of the equations.
Figure 3. Fitting between tide measured and simulated values for Montevideo.

Figure 4. Tidal currents and levels at Torre Oyarvide.
Figure 5. Numerical simulation of the Rio de La Plata (allowing for the total outflow of the Parana and Uruguay rivers and the Coriolis effect).

\[
\begin{pmatrix}
M_{j}^{11} & 0 & M_{j}^{1e} \\
0 & M_{j}^{11} & M_{j}^{1e} \\
0 & 0 & M_{j}^{11}
\end{pmatrix} \times \left( \begin{pmatrix}
\frac{\partial U_j}{\partial t} \\
\frac{\partial V_j}{\partial t} \\
\frac{\partial \xi_j}{\partial t}
\end{pmatrix} 
\right)
\]

\[
\begin{pmatrix}
M_{j}^{21} + M_{j}^{2e} + gM_{j}^{11} - \frac{\epsilon}{\rho} M_{j}^{1e} \\
-jM_{j}^{11} \\
M_{j}^{21} + M_{j}^{2e} + gM_{j}^{11} - \frac{\epsilon}{\rho} M_{j}^{1e} \\
0
\end{pmatrix} = 
\begin{pmatrix}
-M_{j}^{11} \\
-M_{j}^{11} + M_{j}^{21} \\
0 \\
0
\end{pmatrix}
\]

where, for example

\[
M_{j}^{11} = \int_{\Delta x} N_j \, d\Delta x
\]  \hspace{1cm} (21)

\[
M_{j}^{1e} = \int_{\Delta x} \left( \frac{\dot{U}_{j}^{2} + \dot{V}_{j}^{2}}{h + \xi} \right) C \, N_j \, d\Delta x
\]  \hspace{1cm} (22)
Figure 6. Numerical simulation of the Rio de La Plata (allowing for the total outflow of the Parana and Uruguay and neglecting the Coriolis effect).

Taking now the contributions of all the elements,

\[
\frac{\dot{\theta}}{\Delta t} + \beta \theta = \bar{F}
\]  

The elementary matrices \( \dot{A}, \dot{B} \) and \( \bar{F} \) were calculated through the Gauss-Legendre integration method for rectangular elements using coordinates of area for triangular elements. The global matrices \( \dot{A}, \dot{B} \) and \( \bar{F} \) were assembled by a technique of skyline storage allowing for the useful height of the columns. In so doing, and for judiciously numbered and dimensioned grids, it is possible to reduce both the allocated space and the CPU time.

Based upon the general recurrence expression for first order differential equations, Equation 25 can be written as,

\[
\begin{aligned}
\frac{\dot{A}}{\Delta t} + \beta B \theta &= \left( \frac{\dot{A}}{\Delta t} + (1 - \beta)B \right) \theta + \bar{F} \\
\end{aligned}
\]  

where \( \Delta t \) is the calculation step and \( \beta \) an interpolation parameter ranging between 0 and 1. The most characteristic values for \( \beta \) are,

\[
\beta = \frac{1}{2} \quad \text{(Crank-Nicolson)} \\
\beta = \frac{3}{5} \quad \text{(Galerkin)} \\
\beta = 1 \quad \text{(Backward difference)}
\]

all of them corresponding to implicit methods (\( \beta > \frac{1}{2} \)).
After performing the operations indicated in Equation 26, the following system is finally arrived at,

\[ \hat{K}\theta = \hat{S} \]  

(27)

The boundary conditions are given by zero normal velocities on the surface and the bottom, as well as by the known velocities and water level at the limits of the domain. At the oceanic boundary, where there are great depths and tide is the dominant forcing, non-linear terms (convection and bottom friction) become negligible. The oceanic boundary condition is, therefore, given by the water level which can be defined through the tide constituents. Fluvial boundaries, on the other hand, require knowledge of the discharge curve of the Paraná and Uruguay rivers.

Once the boundary conditions had been imposed, system Equation 27 was solved through a direct method adapted to non-symmetrical positive defined matrices based on the Gaussian elimination algorithm. The results obtained are the mean velocities over the water depth and the water level at every point of the finite-element grid at any time. The system of equations was solved as many times as was required by the number of calculation steps.

In its most general version, the model allows for whatever law regarding water level and currents at the boundaries, variable roughness, the Coriolis effect, variable winds and atmospheric pressure, salinity gradients, and variable boundaries. The model has shown a quite stable behaviour and has so far made the use of artificial numerical diffusion unnecessary ($\epsilon = 0$).

Calibration

To calibrate the model, 15-day water level records were selected from La Paloma, Montevideo, Arrospide, Colonia, Buenos Aires, Torre Oyarvide, and Pinamar (Figure 1), and subjected to Fourier analysis and sinusoidal regression. Figure
3 shows that the fitting between measured and simulated values for Montevideo is quite good with average errors of about 4% of the tide range. Meteorological effects were calibrated separately using 7-day hourly wind records from nine stations along the Rio de La Plata.

To verify that the global model was able to generate suitable boundary conditions for the local model (Port of Montevideo), current measurements were performed at ten stations for four months with an ANDERAA RCM-4 current meter. Figure 4 illustrates typical tide and current records obtained at Torre Oyarvide. The calculated tide is superimposed on the diagram and the current resolved into two directions of maximum (Vel x) and minimum (Vel y) variation. Figure 4 shows that there is a dominant runoff direction. The results of the calibration were satisfactory with average fitting errors of about 7%. Tide and wind calibration were carried out using 30–60-minute calibration steps, respectively, with a CPU time of 56 seconds. A 24-hour period was adopted for the tide, which required a CPU time of 45 minutes for each assay.

RESULTS AND DISCUSSION

Once the model had been calibrated the following numerical experiments were performed:

1. Analysis of its sensitivity to the outflow of the Paraná and Uruguay rivers, with (Figure 5) and without the Coriolis effect (Figure 6).
2. Tidal wave progress simulation for a 15-day period representative of the mean year.
3. Simulation of wind action and storm-surge propagation from the continental shelf for a 7-day period representative of the mean year.

As regards to the basic hydrodynamic regime of the Rio de La Plata, the results obtained may be summarized as follows. In the outer region, the circulation pattern is very complicated due to the difference in range and epoch of the main tidal...
constituents as well as to the great relative importance of the Coriolis effect.

On these conditions, significant cross currents arise between both banks up to the beginning of the intermediate zone (Montevideo-Punta Piedras), where the circulation becomes fairly regular because of the bottom and coastal topography (Figures 7, 8 and 9). The main flow concentrates between the southern talus of Banco Ortiz and the Argentine coast.

Fresh water prevails over the whole extent of the inner and intermediate regions up to El Codiillo as a result of the strong fluvial influence. Comparative analyses of residual currents in the outer region show that fluvial waters run through two main tongues, the most important of which flows northwards between Banco Inglés and the Uruguayan coast until reaching the ocean (Figures 10 and 11).

This basic hydrodynamic regime is, however, strongly influenced by meteorological forcing, which causes the water level to be remarkably changed in the inner and intermediate regions and affects the currents in the outer region.

To analyse this meteorological influence, the time series for the water level were subjected to filtering techniques, and the residual series correlated along the Río de La Plata. In so doing, it was found that the meteorological effects are due not only to wind blowing over the whole surface of the river, but to long period storm-surges coming predominantly from the southern sector of the continental shelf and propagating into the river. Thus, in order to simulate the meteorological conditions adequately, it is necessary to take account of the wind stress on the whole surface of the Río de La Plata and to also of the sinusoidal components in excess of 24 hours associated with weather conditions along an oceanic line joining Pinamar to La Paloma. Figure 12 displays the correlations obtained between Pinamar and Torre Oyarvide, and between Torre Oyarvide and Bue-
nos Aires, with phase differences of +8 and +5 hours, respectively. These correlations cover the outer and intermediate areas and proved to be satisfactory.

Additionally, the available winds statistics show that winds in the Río de La Plata are characterized by 2 to 4-day cycles and blow predominantly from the E, N and SE. From the viewpoint of the hydrodynamic regime, southeasterlies are very important, for they have the greatest average intensities and associated energy, and blow almost along the river axis.

The results of wind action simulation can be summarized as follows. Northwesterlies produce a quick response all over the Río de La Plata. Their influence add to the discharge of the rivers Paraná and Uruguay, strengthening the outgoing currents and causing the water level to fall. Within the intermediate area, the outflow becomes regular and parallel to both banks, though more intense along the Argentine coast. On reaching the outer region, the outflow bends towards the Uruguayan coast due to the Coriolis effect and divides into two branches, the most important of which flows between Banco Inglés and the coast. Wind action strengthens particularly the southern branch between Banco Inglés and Banco Rouen. In Bahía Samborombón, the currents are weak and rotate clockwise.

Southeasterlies, on the other hand, bring about a very quick response against the fluvial flow in the outer zone. About 10 to 12 hours after the southeasterlies have begun to blow, progressive ingoing currents develop in the Río de La Plata until reaching the inner region. From that moment, the wind stress cannot balance the pressure gradient force due to the piling-up of water, and the current begins to reverse in the inner region well before the southeasterlies stop blowing.

The characterization of the hydrodynamic regime of the Río de La Plata was completed through a statistical analysis of the results from the sim-
ulation of tide and wind effects along their representative periods for a mean year (Figures 10 and 11). Clearly, the hydrodynamic effects due to the tide and winds are nearly of the same magnitude. It should be noted the remarkable influence of wind action upon the residual currents.

CONCLUSIONS

The Rio de La Plata is a shallow water environment whose hydrodynamic regime is basically determined by the tide and the discharge of the Paraná and Uruguay rivers. Wind action, however, modifies this regime in such a way that the resultant hydrodynamic situation at any point is given not only by local conditions, but also by the global response of the whole system to the actual driving forces. This makes boundary conditions difficult to define through field measurements alone. Under these circumstances, there would seem to be no choice but to develop a global model of the Rio de La Plata for determining the typical hydrodynamic situations and furnishing local models with the necessary boundary conditions. Keeping in mind all these factors, the following conclusions may be drawn:

(1) Nested models—local and global—should be used to characterize the hydrodynamic regime in a particular area.

(2) Any global model requires in situ measurements at the oceanic boundary to have a good knowledge of the range and epoch of each tide constituent.

(3) A database should be set up to store data regarding the physical variables that determine the hydrodynamic regime of the Rio do La Plata (water level, currents, winds, salinity, river discharge, bathymetry, etc.). These data will be necessary if very small grid mathematical models are to be developed in the future.
Figure 12. Tidal correlation between Torre Oyarvide-Palermo and Torre Oyarvide-Pinamar.
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LITERATURE CITED


