A Field Data Assessment of Contemporary Models of Beach Cusp Formation

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ABSTRACT


Cusp formation was observed during an instrumented, daily profiled, time series of a reflective beach in Canaveral National Seashore, Florida on January 5, 1988. The monitored cusp embayment formed by erosion of the foreshore and the cusp series had a mean spacing of approximately 28 m. During this time, inshore fluid flows were dominated by two standing edge waves at frequencies of 0.06 Hz (primary) and 0.035 Hz (secondary) whereas incident waves were broadbanded at 0.12-0.16 Hz. Directly measured flows (and indirectly estimated swash excursion) data support both the standing wave subharmonic model (GUZA and INMAN, 1975) and the self-organization model (WERNER and FINK, 1993) of cusp formation in this study.

ADDITIONAL INDEX WORDS: beach cusps, foreshore changes, edge waves, swash excursion.

INTRODUCTION

Recently, Werner and Fink (1993) successfully simulated the growth of beach cusps through feedback between alongshore variation in foreshore topography and swash kinematics. The role of swash excursion length in the dimensional development of cusps had been proposed by Longuet-Higgins and Parkin (1962) and hence by Dean and Maurmeyer (1980) but the physics coupling uprush length with cusp spacing were not evident. This self-organized flow-morphology model is suggested to be physically incompatible with the widely held concept that cusp spacing and form are due to the presence of alongshore standing waves (Guza and Inman, 1975; Huntley and Bowen, 1978; Sallenger, 1979). Standing waves are usually thought to be either synchronous or subharmonic (Guza and Davis, 1974; Guza and Inman, 1975) edge waves forced by incident waves on steep, reflective beaches and, indirectly, may even result in a distinct sedimentologic expression (e.g. Sherman et al., 1993). Both the subharmonic and self-organized models of cusp formation predict similar spacing over a wide range. Both theories are simple and based upon monochromatic wave conditions and thusly are not explicitly applicable to natural environments where a spectrum of wave motions and a variety of slopes and sediment textures are usually present, i.e. the models are deterministic but the field measurements must be treated statistically. Still, field data supporting each model exists (Dean and Maurmeyer, 1980 for self-organization; Guza and Bowen, 1981 for standing waves).

There is also a morphogenetic problem with cusp formation. Most investigators believe cusp bays form by erosion of the foreshore but Dubois (1981) has observed cusp formation during a foreshore accretion phase. The self-organization model (Werner and Fink, 1993) argues that topographic lows may deepen via backwash acceleration to form cusp bays or topographic highs may grow into cusp horns through run-up deposition. Numerous other factors have been invoked which may cause a periodic channeling of the backwash and form a cusp bay (see review by Komar, 1983). Nevertheless, Williams' (1973) conundrum on “the problem of beach cusp development” remains unresolved after twenty years of sporadic study.

We conducted a reconnaissance study of surf-zone processes and beach change in 1988 and 1989 during which cusps formed and disappeared along the axis of a beach survey line, and while we had instruments deployed. Although our focus was directed towards other, more general, aspects of beach morphodynamics, we have documented that the complex structure of nearshore motion at the site includes standing edge waves (Allen et al., 1991; Bauer and Allen, 1995). The recent discussion of cusp development by Werner and Fink (1993) identifies relevant variables consistent with our data sets. This allows us to present additional field data to evaluate the applicability of each cusp model.

STUDY SITE

Our study was part of a larger investigation of beach/dune dynamics at the north end of Canaveral National Seashore on the central, Atlantic coast of Florida, USA (Allen, 1991). This coast has a semidiurnal tide with a mean range of 1.2 m (which occurred during our visit). The modal beach state
of the site is a longshore bar-trough form, with a locally steeper foreshore and coarser sediments than found nearby. Presumably, this results from being in a zone of convergence in a cellular littoral drift pattern with no net directionality (STAPOR and MAY, 1983). The mean sediment grain size on the mid-foreshore was 0.47 mm (1.10 φ) in our study. The study site is located immediately south of a rip-rap revetment. During energetic wave states, there is a primary breaker line at an offshore outcrop of coquina rock, presumably the Anastasia formation (STAUBLE and DACOSTA, 1987).

STUDY DESIGN

Daily beach profiles were surveyed perpendicular to shore from a local benchmark using a surveyor's level, stadia rod, and tape at the same site from January 4 to January 8, 1988. This local monument was tied to the benchmark for R-208 of the Florida Department of Natural Resources to obtain a vertical datum reference. Offshore wave data are gathered regularly by a state-ringed series of pressure gages operated by the University of Florida. Unfortunately, the nearest one working during our study was the Marineland gage, located 80 km to the north in a depth of 10 m, and has limited relevance. Local nearshore fluid motions were monitored by three packages measuring fluctuations in water depth and cross-shore and longshore flow velocities. Co-located Marsh-McBirney 511 bi-directional, electromagnetic current meters and pressure transducer wave gages were deployed in a variety of arrays during each day. The sensors were cabled to shore where a Sea Data 1255B data logger sampled the analog signal on each channel 2048 times at 2 Hertz, usually with a hourly burst interval. An idealized plan view of the beach configuration, profile line, and instrument sites for the morning of January 5 are given in Fig. 1. All instruments in the alongshore array were located 2 m seaward of the fore-

Figure 1. A: Location of study site in Florida, USA (DB = Daytona Beach, O - Orlando, T = Tampa, M = Miami) B: Diagrammatic view of beach configuration and profile line 6 at the northern boundary of Canaveral N.S. with instrument sites for data run at 1204, 5 January 1988. Florida DNR monument R-208 is actually west of the offset reference location.

Figure 2. Beach profiles for 4-6 January 1988 with focus on foreshore/nearshore change.
quite regular at the study site but they disappeared approximately one kilometer southwards; there were none to the north.

Wave Field

The offshore wave field measured at Marineland on January 4 had significant heights of 0.8–0.9 m with peak periods of 8–9 sec continuing through midnight. The next day, when cusps appeared, wave heights increased to 1.23 to 1.45 m but with a shorter peak period in the 6–7 sec band. On the following day, when cusps had disappeared, wave heights remained at 1.2 m with a peak period of 4–5 sec. The Florida gage data are given in a form of per cent total energy for various wave period classes; these can be reformatted into a synthetic energy spectrum partitioned into variable-width frequency bands which reveal no substantially higher frequency peaks. The offshore waves shoaled into shallower water where they had visually estimated heights of 2 to 2.5 m at the first break and 1 m at another breakpoint on the longshore bar face. Wave heights were lower in the afternoon due to depth limitation near low tide (HT was +1.2 m about 0930 and LT was 0.0 m above MLT about 1600 on January 5).

During post processing we found that many of our inshore records are excessively noisy during cusp formation but the run obtained at 1204 on January 5, on the mid-ebb tide, is clear for several locations. However, water surface elevation measurements, taken approximately where wave peaking took place but seaward of the final breakpoint, are limited to a Paros gage coincident with the current meter located 2 m seaward of foreshore toe at the south horn of the cusp. This gage measured a mean depth of 1.5 m, a significant height of 0.48 m, and a peak spectral period of 12 to 17 sec (a frequency band of 0.06 to 0.08 Hz on Fig. 3). There are two secondary peaks, a subtle, higher frequency one at 0.12 Hz corresponds somewhat with the offshore peak period and a significant, lower frequency peak at 0.03 to 0.04 Hz.

Horizontal Velocity Field

Mean flows measured 15 cm above the bed at the three sites during the same measurement record are given in Table 1 and are directionally consistent with weak offshore flow and a southward longshore flow, strongest in the bay in both cases. The standard deviation in the bay was 50% greater than at a horn. During the 6 hours of deployment (1035–1640) of the inshore current meter, the cross-shore flow varied from −0.055 to −0.092 m/sec and the longshore flow varied from −0.143 to −0.221 m/sec. At 1102, we obtained a spatially-averaged estimate of the longshore current in the nearshore trough, slightly seaward of the current meters, by timing the movement (southward) of a dye plume over 36 m in 44 sec (0.8 m/sec).

Spectral analysis of the oscillations in cross-shore flow yields a narrower peak at 0.06 Hz than found in the surface motion spectrum, although in the same range, and this occurs at both horns and in the bay (Fig. 4). A secondary but significant peak in these cross-shore velocity spectra occurs at 0.12 to 0.14 Hz, close to the frequency reported for offshore incident waves and is more clearly defined than in the pressure spectrum. Little pressure energy is found at 0.04 Hz but clearly there is relatively more energy in the cross-shore flow spectrum where it forms a shoulder on the low frequency side.

Table 1. Nearshore flow means* (and standard deviations) for record 88-1-5-1204

<table>
<thead>
<tr>
<th></th>
<th>North Horn</th>
<th>Bay</th>
<th>South Horn</th>
</tr>
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<tbody>
<tr>
<td>Cross-shore (m sec⁻¹)</td>
<td>−0.027 (0.15)</td>
<td>−0.065 (0.23)</td>
<td>−0.015 (0.16)</td>
</tr>
<tr>
<td>Longshore (m sec⁻¹)</td>
<td>−0.122 (0.09)</td>
<td>−0.19 (0.13)</td>
<td>−0.131 (0.09)</td>
</tr>
</tbody>
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*Negative signs for mean flows indicate directions seaward and southward, respectively
of the dominant peak. There is an apparent reversal of this pattern at 0.09 Hz where a cross-shore flow spectral “valley” is not in accord with the spectral signature of surface elevation variability.

**DISCUSSION**

**Standing Wave Model**

Cross-spectral phase relations of the $\eta-u-v$ triad at the southern horn instrument package (Fig. 5) provide unambiguous definition (Huntley and Bowen, 1978) of progressive gravity waves at the incident frequency band of approximately 0.12 to 0.14 Hz and two standing edge wave structures. One of the latter is consistently and significantly coherent at about 0.06 to 0.07 Hz and a less well defined structure is found at 0.035 to 0.045 Hertz. Other significant peaks in coherence shift substantially in frequency or do not possess the 90° phase angles of a standing edge wave. The cross-spectral coherence signature for $u-v$ in the bay (Fig. 6) differs substantially from that at a horn because of the large amount of coherent energy at 0.07 to 0.09 Hz but, also, remains high at 0.045 Hertz; the phase angles of zero for both peaks are consistent with that of a standing edge wave.

A detailed resolution of the longshore structure of these standing waves is hindered by our inability to apply Huntley and Bowen’s phase angle model at all of our few sites, which would allow for a quasi-objective identification of the wave type within the full frequency domain. However, Sallenger and Holman (1984) argue that “peaks and valleys” in the spectral signature of instrument data are useful in evaluating the structure of standing waves in the nearshore, although the spectra shape indicates the presence or absence of wave motion at those frequencies at that instrument position only. Other instrument sites may have different spectral shapes and this spatial variation is difficult to interpret but useful. The spectra for an instrument located at a standing wave node would possess low energy for surface motion but high flow energy whereas an antinodal site would exhibit high surface motion energy but little flow energy at the same frequency of the peak/valley relationship. These fundamental properties of standing waves are especially useful to interpret our limited data sets describing nearshore motions close to the cusp bay and horn where more formal cross-spectral analysis is precluded. Whether the standing edge wave is synchronous or sub-harmonically forced by incident waves, its longshore structure should have an antinode at the bay and a node at the horn. For the record obtained at 1204 on January 5, the lack of energy in $\eta$, combined with relatively high

Figure 5. Cross-spectral coherence and phase relations of A: $\eta-u$ and B: $\eta-v$ at the south horn (BW = 0.0186 Hz, 25 DOF). The 0.95 significance level of coherence is 0.25.
energy in $u$, at 0.04 Hz argues for the horn being associated with a nodal location in the longshore structure of the standing wave. There is less, albeit insignificantly so, energy in $u$ at 0.04 Hz in the bay than at the horn which might be interpreted as being closer to an antinode. Similarly, the far greater (and significant at the 0.95 level) coherence of $u$ and $v$ at 0.06 to 0.08 Hz in the bay than at the horn, argues for antinodal dominance of the cusp bay. The relationships are approximately the same for the north horn but, because we are unable to assess two-thirds of the triadic interaction at that site as its pressure gage did not function, we rely on the southern horn as a more robust estimator of flow structure.

The longshore wavelength $L_\eta$ of a standing edge wave is given by URSELL (1952) as:

$$L_\eta = \frac{\gamma^2 T^2}{2\pi} \sin(2n + 1)\beta$$

where $T$ is the period and $n$ is the edge wave mode. For $n = 0$ and using a slope of 0.119 (6.8°), the longshore wavelengths for our frequencies of interest delineated by the peaks in the pressure spectrum centered at approximately at 0.03 Hz ($T = 33$ s) and at 0.075 Hz ($T = 13.3$ s) are 202 m and 33 m, respectively. There is, however, substantial sensitivity to the frequency chosen. For example, if the frequency for the spectral peak in cross-shore flow at 0.06 Hz ($T = 16.7$ s) is substituted, then $L_\eta$ is 51.6 m. The cross-shore flow variance is maximised at the antinodal position and is least sensitive to instrument sitting being slightly off the exact location of the antinode relative to instruments and the node off of the horn, so the standing, longshore wavelength 51.6 m is favored. Non-zero edge wave modes have been documented at the site at other times (ALLEN et al., 1991; BAUER and ALLEN, 1995) but would result in much longer standing waves.

Cross-spectral analysis between sites in the alongshore array adds some support for the presence of a standing wave structure although equipment limitations precluded a simultaneous cross-shore array which may have reinforced such an interpretation. The lack of significant coherence at 0.06 Hz in both longshore and cross-shore flows between the bay and the south horn instruments (Fig. 7) is theoretically consistent for nodal-antinodal cross-spectra. Significant, in-phase, cross-shore flow coherence, centered at 0.035 Hz and bounded by phase jumps, is also apparent in the bay-horn relationship. This is reasonable because the corresponding standing wave length is four times the length for the 0.06 Hz structure so its nodal-antinodal positions are substantially different from the higher frequency waveform. Thus the bay-horn instrument cross-spectra are strongly coherent because neither are near a nodal point. Additionally, the two structures are sufficiently close in frequency resolution to consider the possibil-

Figure 6. Cross-spectral coherence and phases of $u$-$v$: A at the south horn and B in the cusp bay ($BW = 0.0186$ Hz, 25 DOF). The 0.95 significance level is 0.25.
Self-Organization Model

WERNER and FINK (1993) claim this model is physically incompatible with the standing wave model yet they predict an equivalent cusp spacing metric. The self-organization model is largely driven by variations in the cross-shore swash excursion distance, S, and that cusp spacing $L_{bc} = fS$ where $f$ lies between 1 and 3 for their simulated cusps. The value of S is largely driven by the positive relationship to incident wave height and period, but the height of shorebreaking waves is limited by nearshore depth. Excursion distance is also controlled by swash/backwash interaction in a complex wave field (e.g. KEMP, 1975). Phase differences between swash and backwash generated by multi-spectral wave fields can either reinforce or retard the uprush hence vary the excursion distance. Our data depict a polychromatic wave field with substantial energy below incident wave frequencies so the excursion distance could not be expressed simply. Clearly, swash motions are characteristically dominated by low-frequency modes of energy that can have a broad array of spatial structures (e.g. HOLMAN and BOWEN, 1982) even when offshore wave spectra are relatively simple. Swash–morphology interaction on natural beaches is also a function of foreshore slope (explicitly included in the simulations of Werner and
Fink) and foreshore-normal variations in grain size and permeability.

The probability distribution best describing a series of discrete swash excursion lengths measured in the field is probably not Gaussian; a skewed distribution is more likely based upon many visual observations of runup by the authors. Holland and Holman (1993) indicate that runup maxima possess a distribution that varies, depending on spectral shape characteristics, but is non-Gaussian for non-narrowbanded spectra. Whether a sample of excursion lengths is best approximated by a Rayleigh (as suggested by Nielsen and Hanslow (1991)) or another distribution is unknown and may vary, depending on local morphodynamic conditions. Neither is it clear what statistic should be used to describe the important distance of swash excursion—and this is partly driven by which probability distribution is used. The apparent choice of a central tendency would be between the median, mean (used by Dean and Maurmeyer, 1980), or root-mean square values of sampled foreshore swash excursion distances. Alternatively, a more extreme measure may be more meaningful physically, perhaps a “significant” distance (i.e. mean of the longest one-third) or a measure at or near the maximum (see Nielsen and Hanslow (1991) for alternative, empirical expressions derived from Australian beaches using the Rayleigh distribution).

We did not measure horizontal swash excursion directly but, using two indirect data sources, estimate a typical value of 17 m for most of the tidal cycle above low tide. Our topographic surveys and notes on drift lines indicate a maximum runup limit during high tide near 15 m from the baseline. The HT location of the shorebreak was approximately 32 m out from the benchmark, the foreshore intercept of mean water level above the step crest (which is at the mean low water elevation). The total distance between the HT runup limit and the beach step at LT is approximately 28 m and the horizontal runup distance would be more than half of this during the upper half of the tidal cycle.

Another source of data on excursion distance is photography of the site taken on 5 January. Photographs of breaking waves and swash uprush maxima were taken at a vehicle barrier composed of a cross-shore series of posts (Plate 1), four cusps to the south of the study site. The posts have known separation distances and thus can be used to estimate the horizontal distance separating the photographed break-point from the photographed uprush location. Although crude, scaling several different photographs suggests a “typical” swash excursion distance of 16 to 18 m at the barrier. We assume the value is representative of the studied cusp as well, located approximately 80 m away.

The measured cusp spacing is thus approximately 1.6 times the estimated horizontal swash excursion distance. The value of our best estimate differs little from their best fit of a slope where $f = 1.7$ in simulations. Werner and Fink present an additional value of the scaling coefficient where $f = 1.6$, obtained from the arithmetic mean excursion distance on natural cusps (Dean and Maurmeyer, 1980) in California, which is in perfect accord with our value derived on a Florida beach.
CONCLUSIONS

During an instrumented, daily profiled, time series of the beach at the north end of Canaveral N.S., Florida, we observed opportunistically the formation of a beach cusp where the bay was eroded into the foreshore, with a horn-to-horn wavelength of approximately 27.7 m. Despite limited instrumentation and survey data, there is statistical evidence (rejection level of $P = 0.05$) for the presence of a subharmonic standing, mode zero edge wave at a frequency of about 0.06 Hz and a longshore wavelength of about 52 m with nodes at the horns and an antinode in the bay. The subharmonic standing edge wave model for cusp formation (Guza and Inman, 1975) fits our cusp data with a coefficient of 1.86, nearly the value of 2 inherent to the model.

Indirect field documentation of a 17 m swash excursion length suggests that the self-organized model of cusp formation (Werner and Fink, 1993) is also in accord with our measured cusp wavelength where $f = f_0 = 1.6$. However, the estimated values, while well constrained, lack precision in evaluating this value's significance. Although we did not directly measure swash excursion, several lines of evidence suggest that the self-organization model is theoretically plausible, physical, possible, and fits our cusp data well. Although the subharmonic model of cusp formation is favored (under the conditions measured) because the physical presence of the required standing edge wave is supported but we cannot dismiss the possibility that the self-organization model was operating also. It is not clear what excursion distance statistic should be used to describe the variable distances encountered on real beaches during broad-band, incident wave conditions yet this is the fundamental control upon any derived, best-fit value of $f$.

Clearly this assessment is not a statistical test of the two models but our field data is more robust than that cited previously and thus reinforces Werner and Fink’s statement (p. 968) that “the two models “are sufficiently similar that current data and observations cannot distinguish unambiguously between them”. Overcoming this will be difficult. A rigorously designed study, specifically (and extensively) instrumented to include offshore arrays and swash excursion lengths, is required to address the present uncertainty associated with the multi-dimensional, cusp formation problem. Guza and Werner attempted this but the initial results were inconclusive (Wakefield, 1994).

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