Least Squares Filtering to Assess Shoreline Change Signatures

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ABSTRACT


This paper discusses an approach to spatial shoreline change rate filtering to effectively improve interpretation of natural shoreline changes and to assist in delineating between human-induced shoreline change and natural fluctuation of the shoreline. A least squares spatial filter method is applied along a section of shoreline in Brevard County, Florida, U.S.A., to assist in assessing effects of a man-made navigation project upon adjacent downdrift shoreline. Results of application of the technique are discussed. Methods utilized by others for temporal and spatial smoothing of shoreline change are also noted.

ADDITIONAL INDEX WORDS: Accretion, beaches, beach width, erosion, mapping, natural system variability, non-stationarity, numerical filtering, shoreline change, spatial change, surveying, temporal change.

INTRODUCTION

Assessment of shoreline change variability with time and space is often a difficult task where natural shoreline trends at various spatial scales are mixed in with man-made shoreline changes due to navigation channels or coastal structures such as jetties or groins. Non-stationarity of shoreline change with space or time provides an additional complicating factor that must be dealt with. Typical shoreline change analyses ignore temporal and spatial “noise” in shoreline change data sets. Most investigators and agencies that regulate shoreline development use a linear endpoint rate method to determine shoreline change rate (White and Morton, 1987; Benton and McCullough, 1988; Foster and Savage, 1989). This method consists at a given location of simply dividing the change (accretion/erosion distance) between two temporal positions of the shore by the time between shoreline surveys (or aerial photographs). One approach to eliminating some “noise” in the temporal change rate at a given shoreline position is provided by Fenster and Dolan (1994), and is referred to as the Minimum Descriptive Length (MDL) method. The MDL method basically consists of a temporal polynomial filter that can be fit to a series of temporal observations at a given shoreline location. Shoreline change rates (at a given time) can then be determined at that location via the derivative of the polynomial filter. The spatial change rate “noise” is more often ignored completely. When not ignored, the engineer or geologist typically deals with the spatial “noise” via a smoothing of shoreline change information spatially using a simple moving average filter of equal weights over a section of shoreline. This procedure is equivalent to averaging the shoreline change rate over a section of shoreline and then plotting that change rate at the center point of the shoreline section. The length of the shoreline section chosen to average over is typically arbitrarily chosen. This simple moving average spatial filter is often applied without concern (or often even knowledge) of what effects such a filtering process will have on the data. It is known though that the effects of applying such a filter can lead to misinterpretation of underlying spatial trends if spatial frequency effects are not understood. As an example of these concerns, it can be shown that the use of Simpson integration for integrating a discrete measured process leads to high frequency noise magnification that can actually make its use less preferable with some data than the simpler trapezoidal integration method. In not so extreme examples, a misapplication of a spatial smoothing filter can introduce non-existent spatial cycles into the smoothed data and consequently provide an incorrect interpreted trend where no such trend exists. The present paper provides an application of the versatile “least squares” filter (i.e. Hamming(1983)) to the filtering of spatial shoreline change rate “noise” from shoreline change rate “signal”. The approach is utilized on a segment of shoreline that has both human-induced shoreline change erosional effects and underlying large natural shoreline change trends. Although separation of the natural and human-induced shoreline change erosional effects and underlying large natural shoreline change trends. Although separation of the natural and human-induced shoreline change signal is not attempted herein, it is clear in the case presented that the natural shoreline change (after the filtering out of the high frequency random noise) has considerable spatial trend that is large in magnitude, sufficiently so to mask human-induced shoreline change trends. Additionally, the filtering approach allows one to more clearly evaluate non-stationarity in data trends.
STUDY LOCATION AND DATA OVERVIEW

The shoreline data utilized in the present study consists of annual shoreline change rates for various time (inter shoreline mapping or surveying) intervals along an approximately 26 mile section of shoreline in Brevard County, Florida. This section of shoreline (Figure 1) lies to the South of Cape Canaveral, Florida, the most prominent coastal feature South of the mid Atlantic coastline capes. From 1951 thru 1954 an entrance channel was dredged and jetties were constructed to provide safe entrance to the newly constructed Port of Canaveral on the Southeast coast of Florida. This area is a zone of net Southward littoral sand transport (Bodge, 1993) and numerous studies have been made to assess the effects of the Port of Canaveral on the downdrift shoreline (i.e. Bodge, 1993; Dean and Work, 1993).

The historical survey data utilized comes from a data base consisting of a number of mean high water shoreline digitized maps developed by the Florida Department of Natural Resources (DNR) (now the Department of Environmental Protection). The digitized maps in turn were derived via digitizing aerial photography or old U.S. Coast and Geodetic Survey (now National Ocean Survey) maps. Details of the accuracy and methodology utilized to convert the aerial and survey data to the digitized database is covered in Demirpolat et al. (1989). The shoreline change rate data used in the present study is calculated from the difference in mean high water shoreline position at two different time periods (i.e. two dif-
different surveys) divided by the difference in time between the surveys. The spatial sampling interval (points at which spatial sampling for analysis purposes was made) for this particular study was approximately 300 meters.

**METHODOLOGY AND RATIONALE**

To ascertain the natural fluctuations of the shoreline change rate (or change), it is helpful to separate high frequency fluctuations of the shoreline change rate which can be considered as “noise” (both measurement noise and small spatial fluctuation) from the low frequency fluctuations of the shoreline change or change rate which can be considered as “signal” due to the superposition of “natural” low frequency shoreline oscillations and a very low frequency signal from effects of Port construction (for those time periods analyzed after Port construction). The very low frequency effect of Port construction can best be conceptualized by viewing of the signal produced downcoast of a complete barrier to sand, which, for simplified purposes, would be the mirror image of the PELNARD-CONSIDERE (1956) solution to the sand transport “diffusion” solution for the case of no bypassing.

After removing high frequency “noise” the remaining signal allows one to compare the low frequency oscillations of shoreline change or change rate for time intervals “before” and “after” Port channel and jetty construction. Assessment of multiple time intervals “before” construction allow one to assess the stationarity of the shoreline change rates. The shoreline change low frequency “signature” between two time intervals (one prior to Port construction and one subsequent to Port construction) might allow for improved identification of the extent of maximum Port induced shoreline recession where “natural” shoreline fluctuation is limited in extent. At a minimum, the low frequency “signal” allows for improved delineation of the levels of “natural” shoreline change that occur along this stretch of shoreline. Additionally, comparison of the low frequency “trends” in shoreline change or change rate prior to other man-induced changes being made allows one to better identify periods of “natural” nonstationarity in the data analyzed. One additional use of the filtering methodology discussed here (but not expanded upon) is in assessment of shoreline mapping accuracy. If numerous inter-survey periods show corresponding shorelines in the same approximate location, but one survey appears to be in far different location (for which there is no apparent reason), then evidence for survey inaccuracy has been uncovered.

The investigative process discussed above was implemented for the Brevard shoreline in a number of steps as follows: (1) development of low frequency “least squares” filter as per HAMMING (1983); (2) application of the filter to the shoreline change and change rates for the purposes noted in the above paragraphs; (3) plotting of the “noise” in shoreline change with distance along the shoreline to assure spatial homogeneity in the high frequency “noise” levels; (4) mapping of the probability density function of the high frequency “noise” and comparison to the Gaussian density curve based on Method of Moment analysis of the “noise”; and (5) plotting the low frequency oscillation in shoreline change and change rate to assess the natural and man-made low frequency shoreline changes and trends. These steps will be expanded upon in the following paragraphs.

**FILTER ANALYSIS AND RESULTS**

The filtering methodology utilized is discussed at length in HAMMING (1983) although a short summary of filter features and its application to the data is rediscussed herein. The filter process consists of fitting a FIR (finite impulse response) moving weight filter to the data. The filter coefficients are designed such that the filter weights when multiplied by the data in the moving filter window best fit a polynomial of a given order (in a “least squares” sense) to the actual data. An example of the general form of the fitting equation for a quadratic least squares filter is given as

$$\hat{u}(x_i) = A + Bx_i + Cx_i^2$$

(1)

where \(\hat{u}(x_i)\) is the filtered shoreline change rate in the present example, with \(x_i\) as the location on the shoreline, and \(A, B, C\) being coefficients to be found. Minimizing the sum of squares of the residuals from the filtering operation gives an equation of the form

$$SSR = \sum (u(x_i) - \hat{u}(x_i))^2$$

(2)

where \(SSR\) = sum of squares of residual from fitting operation. Upon differentiating this equation with respect to the unknown coefficients and solving the resulting system of equations for \(A, B, C\) the filter expression can be found. Typically, the filter is centered and the spatial scale is normalized leading to a set of normalized filter weights which only requires for solving the resulting system of equations for \(A\). For the present example with a 5 point quadratic filter a filtered point would be

$$\hat{u}(x_i) = [ -3u(x_{i-2}) + 12u(x_{i-1}) + 17u(x_i) + 12u(x_{i+1}) - 3u(x_{i+2}) ]/35$$

(3)

As results of various polynomial orders and various lengths of window provide different transfer functions of the filter, various implementations of the filter can be tried to obtain the desired level of noise reduction. The best filter can be judged by comparison of the filtered data to the original data and by comparison of the spatial homogeneity of the resulting passed data. In the present study polynomial orders of two thru four were tried while window lengths of approximately 3,000 to 12,000 meters were tried with the filter orders. Use of the filter with simulated data has shown that the filter order used and the data window length used is by necessity driven by the characteristics of the data with shorter window lengths and higher order polynomials used with more pronounced shoreline features. In the present study the polynomial order of four and data window length of approximately 6,000 meters was found to smooth the shoreline and yet retain the characteristics of the low oscillations in the shoreline. Figure 2 provides the transfer function of the filter utilized in the present study. The weights of the filter utilized are shown in Figure 3.

Application of the filter was made to four intersurvey periods where the shoreline change rate in meters/year was the process being filtered. Three of the intersurvey periods were
for periods prior to construction of the Port while one intersurvey period after Port construction was filtered. For the period prior to Port construction, two of the three intersurvey periods filtered were adjacent but independent time periods, while the third intersurvey period encompassed the two others. The time periods covered were: (1) Prior to construction: 1876–1928, 1928–1947, and 1876–1947; and (2) Post construction: 1947–1969. As construction of the Port initiated in 1951 the post port construction data interval was assumed 1951–1969. An example plot of both the measured annual shoreline change rate and the filtered shoreline change rate for the post Port construction period is given in Figure 4 to show the characteristics of the filtering operation.

Plots of the filtered annual shoreline change rates for the three intersurvey periods prior to construction are shown in Figure 5. It is apparent in this Figure that the two independent intersurvey periods, 1876–1928 and 1928–1947 are clearly very different in their trends of accretion and erosion. The earlier intersurvey period shows a more dominant trend of accretion near the Port location (prior to Port construction) and a relatively stable \( \frac{0.5}{\text{year}} \) shoreline further away from the Port location. The latter intersurvey period shows low frequency oscillations have developed in the shoreline. The intersurvey period which encompasses the entire time span, 1876–1947, again looks similar to the earlier intersurvey interval due to the longer time interval represented by the earlier intersurvey period. It should be noted that po-
Figure 6. Filtered Shoreline Change Rates Prior and Subsequent to Port.

Figure 7. High Frequency Shoreline Change Rate Noise (1947–1969).

Figure 8. Probability Density and Gaussian Fit to H.F. Noise (1947–1969).

tential bias in the location of the mean sea level of the 1947 data (Foster, 1994) does not allow for reliable mean rates of shoreline change in intersurvey periods that include this data. For this reason intersurvey periods that include the 1947 data must be interpreted based on relative spatial changes rather than absolute rates. It is clear from this Figure that the pre Port shoreline change trends appear to be non-stationary and that interpreting the more recent events by the earlier data or even the entire data time period could be very misleading.

Figure 6 provides the filtered shoreline change rates for the period just prior to Port construction and just after Port construction. In both the prior Port intersurvey period and the post Port intersurvey period large low frequency oscillations in shoreline change rates are apparent. In addition, the low frequency oscillations are out of phase with each other, therefore providing large shoreline change rates for given localities along the shoreline. These large change rate differences between intersurvey periods are much larger that any average change rate for the shoreline. Except for the near vicinity to the Port, these low frequency oscillations in the shoreline change rates considerably mask any overall signal due to Port construction which would show up as a decreasing (with distance from the Port) erosion rate under the scenario of an assumed net Southward littoral drift.

An example of the characteristics of the high frequency noise that have been disregarded in this analysis are as shown in Figure 7 which is for the post Port construction period. It is clear that there is no spatial inhomogeneity in the shoreline noise for the filter utilized. It should be noted that since the filter requires data on either side of the filtered point, the first few points are zero. As the plot does not encompass the entire shoreline analyzed, similar zero noise values are not shown at the distant end. In all cases, similar trends were seen in the spatial homogeneity of the high frequency noise. A probability density function of the high frequency noise for the same set of data is shown in Figure 8 where a Gaussian frequency curve has been fit to the data to show that the high frequency noise can be considered Gaussian in nature. The reason for the high bin value at zero is due again to the end points of the filtering process producing zero noise which was not taken out in this analysis. Similar trends for a Gaussian distribution of the high frequency noise were again seen for all intersurvey periods analyzed.

**SUMMARY**

Results of the above investigation show an effective filtering technique for eliminating high frequency noise from
shoreline trend analysis and clarifying shoreline change rate analysis. Comparison of filtered shoreline change rates show that shoreline change rates may have low frequency oscillations in them that mask other non natural signals and even exceed by a large measure the mean trend of the shoreline. High frequecy noise in the shoreline change rates is noted to be reasonably Gaussian and spacially homogeneous. Non-stationarity in shoreline change rates can provide complications to shoreline trend analysis.

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LITERATURE CITED


