A Fractal Approach to Sea-Surge Occurrences in the
Northern Adriatic Sea

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ABSTRACT


The underlying complex physical processes associated with the generation of sea-surges in Venice and in Trieste, located in the Northern Adriatic Sea, are found to be sufficiently scale invariant in respect of their magnitude-time patterns. This provides a rational basis (i) for the computation of the relative sea-surge risk normally underestimated when it is obtained through the classical exponential distributions and (ii) for the identification of fractal time clustering patterns not obtainable from a simple linear approach.

ADDITIONAL INDEX WORDS: Sea-surge, fractals, Cantor dust.

INTRODUCTION

The genesis of the sea-surges in the Northern Adriatic Sea is very complex depending on different forcing factors that, interconnected among themselves, are responsible for large increase in sea level with dramatic consequences for the sea cities, vegetation, aquifers, estuaries and sedimentation. The difficulty in finding a simple linear relationship which links all these different factors is the main reason to follow a non-linear approach (Mandelbrot, 1983). The surges in periods of maximum sea level have a greater probability of turning into flooding. An application to the sea-surge occurrences in the cities of Venice and Trieste, located in the Northern Adriatic Sea and subjected to frequent sea-floodings, is provided. Extreme annual sea-surges and the relative times of occurrence recorded in Venice from 1872 to 1995 (Pirazzoli, 1982, and updates) and in Trieste from 1875 to 1995 (Ferraro, 1972, and updates) are analysed. The values in Venice are referred to the mean of low and high tides occurred during the 1884-1909 interval, while those of Trieste are referred to 399.3 cm under the benchmark of the National Geographic and Geodetic Service. The distribution of sea surges in Venice and in Trieste is found to be sufficiently scale-invariant in respect to magnitude and times of occurrences. This provides a rational basis for the computation of the sea-surge risk normally underestimated when it is obtained through the classical exponential distribution.

CAUSES OF SEA-SURGES IN THE NORTHERN ADRIATIC SEA

Relatively small increases in sea level could greatly expand the land area subjected to flooding, and thus increase the coastal hazard. The factors responsible for the Northern Adriatic sea-surges are here summarized.

(1) When low pressure passes over the western and central Mediterranean (and particularly over the Ligurian or Tyrrhenian Sea) and then moves toward the north-east generates the Sirocco wind which blows from SE along the main axis of the Adriatic Sea and this is the main source of transport of sea water. This adverse situation may last for several days when a cyclonic depression is generated in the lee of the Alps over the Ligurian Sea, or in the cold season, when the sea waters are warmer than the air masses and thus generate cyclogenesis over the Ligurian Sea by supplying the air masses with a large amount of heat and moisture. The aerodynamic interaction between the Sirocco and the Alpine chain generates a "Dark Bora" (i.e., a fresh wind from NE associated with heavy rainfall which blows below the Sirocco flowing through a gate in the Dinaric Alps, and this causes the additional transport of sea water across the minor axis of the basin. When the Sirocco blows, the rise Dh in sea level ranges from Dh = 20 cm with a wind-speed of 10Km/h to Dh = 100 cm with a wind-speed of 60Km/h (Caloi, 1973); the Bora is responsible for approximately half of this rise.

(2) The Adriatic is a semi-closed sea, and a non-homogeneous distribution of the atmospheric pressure over its surface causes a rise in sea level when the pressure is lower; i.e., as 10.33 m of water corresponds to 1013hPa, in a space gradient of pressure, 1hPa depression causes, locally, the sea level to rise by almost 1 cm. The usual range of Dh induced by this forcing is 10 < Dh < 20cm, although an upper limit Dh = 45cm may be possible. The pressure pattern over the Mediterranean varies daily, but some weather types are more recurrent in certain seasons.

(3) Rapid changes in the atmospheric pressure pattern cause
free oscillations in the Adriatic Sea (named “seiches”) where the first two resonant periods are of 23 hours and 11.7 hours, respectively both close to the dominant tidal diurnal and semi-diurnal periods, and the seiches may last for several days always keeping the same phase (CALO, 1973). The average rise in the level of the sea caused by the seiche is Dh = 45 cm, although the maximum expected value is double. It was very lucky that, during the extreme surge occurred in Venice in 1966, the seiche was in opposition of phase with the tide, otherwise the situation would have been extremely critical for the survival of the city and its inhabitants.

(4) The tide in the Adriatic is a mixed tide, the semi-diurnal and diurnal components being the most important followed by the fortnightly (14.28 days) and nodal (18.6 years) modes (MAZZARELLA and PALUMBO, 1994). In this century, the astronomic lunar-solar factor may be responsible for Dh = 25–80 cm.

(5) The mass balance among rainfall, evaporation and the processes, fractal distributions model processes that exhibit scale invariance (self-similarity). If the number of objects N with a characteristic linear dimension greater than r satisfies the relation:

\[ N = C r^{-D} \]  

a fractal distribution is defined, where C is a constant of proportionality and D is the fractal dimension that provides a measure of the clustering of the objects versus r. The more isolated the clusters the smaller the value of D. The basic concept of a fractal distribution is that a phenomenon will be repeated on different scales in the same manner, the major variable being the fractal dimension which is used as a measure of the nature of the phenomenon. D is always higher (provided it is a fractal set) or equal (if it is not a fractal, but a simple Euclidean subset) with the topological dimension DT of the set where DT = 0 (point), DT = 1 (line), DT = 2 (surface), DT = 3 (space) etc, for higher dimensions. Also, D is lower or equal to topological dimension of metric space in which the set belongs (Mandelbrot, 1983; Luongo et al., 1986a; 1986b). The interval t, over which the series of N events occurs, is divided into a series of n smaller intervals t = t/n with n = 2, 3, 4... and the fraction Pr = N/n of intervals of length t occupied by events is computed. If the distribution of events has a power-law structure then one obtains:

\[ \log (N) = \log (C) - b \log (m) \]  

\[ N = C m^{-b} \]

where C is a constant of proportionality. The specific limits, inside which N is related to m according to an exponential law, are obtained verifying the confidence level of the relationship:

\[ \ln (N) = \ln (C) - bm \]  

When the sea-surge magnitudes are treated as continuous functions, the fractal dimension D can be obtained through the exponent b (Turcotte and Green, 1993):

\[ D = 2 - 1/b \]

A scale-invariant distribution of sea-surge can be also expressed in terms of the index of severity for future sea-surges:

\[ F = 10^{0.5 \text{Pr}} \]

(Turcotte and Green, 1993): the higher the value of F the more likely that severe sea surges will occur.

Sea-Surges and Time of Occurrence

The fractal relationship of time distribution of sea-surge magnitudes verified to be scale invariant is investigated on the basis of the Cantor dust model (Mandelbrot, 1983; Luongo et al., 1986a; 1986b). The interval t, over which the series of N events occurs, is divided into a series of n smaller intervals t = t/n with n = 2, 3, 4... and the fraction Pr = N/n of intervals of length t occupied by events is computed. If the distribution of events has a power-law structure then one obtains:

\[ \log (Pr) = \log (C) + (1-D) \log (t) \]

or, equivalently, on a log-log scaled plane:

\[ \log (Pr) = \log (C) + (1-D) \log (t) \]

where C and D are constants. D is estimated from the regression coefficient of linear relationship (5) when it is found to be confident at not less than 99% and to deviate signifi-
Fractal Approach to Sea-Surge Occurrence

RESULTS

Sea-Surges and Magnitude

In calculating the cumulative frequency distribution in the log-log plane, the values of the yearly extreme values of sea level are grouped into intervals which are sufficiently large to account for the error due to the method of estimating the sea level height. After preliminary tests, an interval of 5 cm yields the best constrained shapes both for Venice and for Trieste data sets (Figure 1) and the best linear fitting occurs for 67 events of magnitude greater than 105 cm in Venice and for 60 events greater than 260 cm in Trieste. The parameters of the exponential (2) and of the fractal relationship (3), both computed at a confidence level greater than 99% level, are reported on Table 1 together with the values of the index F computed according to (4). The values of the mean periods of return computed according to power-law relationship for some damaging sea-surges in Venice and in Trieste are found systematically shorter than those computed according to exponential one.

Sea-Surges and Time of Occurrence

The Cantor dust model is applied to the 67 sea-surges in Venice and to the 60 sea-surges in Trieste whose magnitude structure is verified to be reasonably scale invariant. The fraction Pr of time intervals including a sea-surge is plotted as a function of the interval size t on a log-log scaled plane (Figures 2, 3). The smallest time interval is chosen to be 1 month. For values of t ranging from 1 to 10 months, Pr in Venice and in Trieste is found to be not significantly different from the correspondent random and uniform simulations; single sea-surges therefore occur in each time interval and no clustering is observed (D = 0); for t > 56 months in Venice and t > 126 months in Trieste each interval contains an event (D = 1); for 18 < t < 56 months a fractal relationship is observed with a fractal dimension D = 0.62 ± 0.01 in Venice and 0.63 ± 0.01 in Trieste, confident at a level greater than 99% and significantly different from the correspondent uniform and random simulations.

DISCUSSION

The complexity of a natural phenomenon does not depend on the number of causes that govern it but essentially on the number of their interconnections, on the magnitude of such linkages and on the feed-back processes. In this case, the whole system is more than the sum of its parts and its re-

Table 1. Values of parameters of power-law and exponential models for sea-surge events greater than 105 cm and 260 cm occurred in Venice and in Trieste, respectively. Here N is the total number of events with magnitude greater than m; b and C are constant; D is the value of fractal dimension and F is the severity index of sea-surge computed according to (4).

<table>
<thead>
<tr>
<th></th>
<th>Venice</th>
<th>Trieste</th>
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<tbody>
<tr>
<td>Fractal Model:</td>
<td>Fractal Model:</td>
<td>Fractal Model:</td>
</tr>
<tr>
<td>log(N) = log(C)</td>
<td>b log(m)</td>
<td>b log(m)</td>
</tr>
<tr>
<td></td>
<td>Venice: log(C) = 19.95 ± 0.33</td>
<td>Trieste: log(C) = 47.52 ± 2.43</td>
</tr>
<tr>
<td></td>
<td>b = 8.89 ± 0.03</td>
<td>b = 18.85 ± 0.98</td>
</tr>
<tr>
<td></td>
<td>D = 1.89</td>
<td>D = 1.95</td>
</tr>
<tr>
<td></td>
<td>F = 1.30</td>
<td>F = 1.13</td>
</tr>
<tr>
<td>Exponential Model:</td>
<td>Exponential Model:</td>
<td>Exponential Model:</td>
</tr>
<tr>
<td>ln(N) = ln(C) - bm</td>
<td>Venice: ln(C) = 11.21 ± 0.24</td>
<td>Venice: ln(C) = 21.59 ± 0.78</td>
</tr>
<tr>
<td></td>
<td>b = 0.065 ± 0.005</td>
<td>b = 0.066 ± 0.005</td>
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Figure 1. Observed cumulative frequencies (star points) with an increment of 5 cm of yearly extreme sea level data in Venice (interval 1872-1995) and in Trieste (b) (interval: 1875-1995). The best fitting regression line of log(N) on log(m) is also reported.

Figure 2. The logarithm of the fraction Pr of time intervals of length t (in months) including the sea-surge events in Venice as a function of log(t). Continuous line links the observed data while the star and dashed lines represent the relative random and uniform simulations, respectively. Vertical dashed lines represent the lower and upper limits of log(Pr) inside which the linear slope provides the best fitting to the data.
response to an external forcing does not follow a linear but a power-law relationship. The sea surges occurring in the Northern Adriatic Sea are due to several forcing factors: a low pressure passing over the Mediterranean and generating a Sirocco wind; the barometric effect associated with a gradient of atmospheric pressure over the sea waters; free oscillations in the Adriatic sea; solar and lunar influences; mass balance among evaporation, rainfall and Atlantic inflow; sea water expansion and melting of continental ice; subsidence of the soil. So, the identification of fractal behaviours in the phenomena is due to the variety of factors interconnected among themselves obey power-law statistics over relatively wide range of scales and, thus, should not be surprising that the variety of factors that determine sea-surge in the Northern Adriatic Sea also produces fractal behaviours with no particular scale connected with their magnitude-time patterns, at least over limited range of scales.

**LITERATURE CITED**


### Table 2. Values of mean return periods provided by power-law and exponential models for different damaging sea-surge intensities recorded in Venice and in Trieste.

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Return period</th>
<th>Power-law Model</th>
<th>Exponential Model</th>
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<tbody>
<tr>
<td>190 cm</td>
<td>243 years</td>
<td>Venice</td>
<td>375 years</td>
</tr>
<tr>
<td>200 cm</td>
<td>385 years</td>
<td>Venice</td>
<td>719 years</td>
</tr>
<tr>
<td>350 cm</td>
<td>232 years</td>
<td>Trieste</td>
<td>365 years</td>
</tr>
<tr>
<td>360 cm</td>
<td>477 years</td>
<td>Trieste</td>
<td>744 years</td>
</tr>
</tbody>
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